Discrete Mathematical Structures

CS 3233 Lecture 31

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Business

- Assignment 11
  - Due Wednesday 11/23 in class
  - Section 4.1: 12, 14, 16
  - Section 4.2: 4, 8, 14
  - Section 4.3: 8, 18, 24

Counting Examples

- How many passwords if they must be 6-8 alphanumeric characters and must include at least one numeric?
  - Idea: all 6-8 character strings minus the ones that contain no numerics
- Internet Addresses
  - Example 15, p307

Principle of Inclusion-Exclusion

- $|A \cup B| = |A| + |B| - |A \cap B|$
- Example
  - How many bit strings of length 8 start with a 1 or end with 00?

Pigeonhole Principle

- Starting Section 4.2
- If k+1 or more objects are placed in k boxes, there is at least one box containing two or more objects
- Proof
  - If each box contains at most 1 object, together they can contain at most k objects

Generalized Pigeonhole Principle

- If N objects are placed into k boxes, there is at least one box containing at least $\lceil N/k \rceil$
- Proof
  - $k(\lceil N/k \rceil - 1) < k((N/k + 1) - 1) = N$
- Example
  - Among 100 people there are at least $\lceil 100/12 \rceil = 9$ people born in the same month

An Tricky Example

- Example 10, p316
  - During a month with 30 days a baseball team plays at least one game a day and not more than 45 games
  - Show there is a sequence of days in this month during which the team plays exactly 14 games

Permuations

- Begin Section 4.3
- Definition
  - Given a set S, an r-permutation of S is an ordered arrangement of r distinct elements of S
- Theorem
  - Given a set S of size n, the number of r-permutations of S is P(n,r) = n(n-1)(n-2)…(n-r+1) = n!/(n-r)!

Examples

- The number of alphabetic strings of length 3 consisting of distinct characters
- The number of one-to-one functions from a set of size r to a set of size n

Combinations

- Definition
  - Given a set S, an r-combination of S is an **unordered** arrangement of r distinct elements of S

Examples

- The number of sets of size three consisting of (distinct) alphabetic characters
- The number of subsets of size r drawn from a set of size n
  - Compared to the set of one-to-one functions from a set of size r to a set of size n, we are considering only the range of the function, not its individual values

A Way of Thinking

- How many ways are there to order a set of size n?
- If you only care about the first r places in the ordering, (n-r)! of the orderings are effectively the same
  - This is because once I've chosen the first r places, there remain (n-r) elements whose order I don't care about
  - Thus, the number of permutations is n!/(n-r)!
  - For the number of combinations, you also don't care about the ordering of the elements in the first r places, so you divide the number of permutations by the number or ways of ordering the first r places, which is r!