Discrete Mathematical Structures

CS 3233 Lecture 25

Prof. William Winsborough

November 2, 2005

Business

- Modification of Syllabus: We will not cover Sections 2.5, 2.6, or 2.7
- Class Schedule
  - The make-up lecture for 10/24, which had been scheduled for recitation 10/31 and 11/2, will be held in recitation 11/2 (today) and 11/7
  - The regularly-scheduled lecture for 10/31 will be made up during recitation period 11/14 and 11/16
- Recall Assignment 8 is due tomorrow: Section 2.4: 2, 4, 6, 14, 16, 28, 30, 40

Greatest Common Divisor

- Let a and b be integers, not both zero
  - The largest d such that d | a and d | b is called the *greatest common divisor*
  - d = gcd(a,b)
- a and b are *relatively prime* if their greatest common divisor is 1

Finding GCDs

- Suppose the prime factorization of non-zero integers a and b are
  $a = p_1^{a1} p_2^{a2} \ldots p_n^{an}$ and $b = p_1^{b1} p_2^{b2} \ldots p_n^{bn}$
- Then gcd(a,b) =
  $p_1^{\min(a1,\ b1)} p_2^{\min(a2,\ b2)} \ldots p_n^{\min(an,bn)}$
- Observe that this value divides a and b, but that no larger value does

Least Common Multiple

- Let a and b be positive integers
  - The *least common multiple* is the smallest d such that a | d and b | d
  - Notation: d = lcm(a,b)
- Using the prime factorization again, lcm(a,b) = $p_1^{\max(a1,\ b1)}\ p_2^{\max(a2,\ b2)} \ldots p_n^{\max(an,bn)}$

Mathematical Induction

- Basics: See Section 3.3
- Induction is valid because the natural numbers are well founded: given any subset, there is a least element
- Once base and step are shown, can derive a contradiction from the assumption that the property fails for some values