Discrete Mathematical Structures
CS 3233 Lecture 33

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November 28, 2005

Business

- Recall from 11/23: Assignment 12
  - Due 2pm Thursday 12/1 at Winsborough's Office
  - Section 5.1: 6, 8, 12
  - Section 5.2: 2, 6, 24, 26, 30
  - Section 5.3: 2, 6
- Any questions from Assignment 11?

Probability Example

- Pennsylvania lottery
  - Player selects 7 integers in [1,80]
  - Commission selects 11 integers in [1,80]
  - Player wins if all 7 are among the 11
- What is the probability of the player winning

Combinations of Events

- Given an event $E \subseteq S$, $\bar{E}$ is the complementary event, S-E. $p(\bar{E}) = 1 - p(E)$
  - If a coin is flipped 10 times, what is the probability of getting heads at least once?
- If E_1 and E_2 are events in S then what is $p(E_1 \cup E_2)$?
  - What's the probability that a positive integer ≤ 100 and selected at random is divisible by neither 2 nor 5?

Probability Distributions

- What happens when all outcomes are not equally likely? Distribution is not *uniform*.
  - Given a finite sample space $S = \{x_1, \ldots, x_n\}$, a real-valued function $p: S \rightarrow [0,1]$ is a *probability distribution* if
    1. $0 \leq p(x_i) \leq 1$, for $i = 1,2,\ldots,n$
    2. $\sum_{0 \leq i \leq n} p(x_i) = 1$
- Biased-coin example: heads twice as likely as tails
- Probability of event E: $p(E) = \sum_{x \in E} p(x)$

Conditional Probability

- A fair coin is flipped 3 times. Assuming it comes up heads the first time, what is the probability it comes up tails an odd number of times?
- Definition
  - Let E and F be events with $p(F) > 0$. The conditional probability of E given F, denoted by $p(E|F)$, is $p(E|F) = p(E \cap F)/p(F)$
- Further examples
  - Suppose that a family has 2 children, one of which is a boy. What is the likelihood that the family has two boys?
  - Suppose that the first child is a boy. Now what is the likelihood that the family has two boys?

Independent Events

- A fair coin is again flipped three times. Does knowing it comes up tails the first time alter the probability that it comes up tails an odd number of times?
- Definition
  - Events E and F are independent if and only if $p(E \cap F) = p(E)p(F)$
- E = a family with 3 children has both sexes
  F = a family with 3 children has at most 1 boy
  Are E and F independent?

Random Variables

- A *random variable* $X: S \rightarrow R$ assigns a real number to each experiment outcome
- The distribution of a random variable X is the set of pairs (r, p(X=r)) for $r \in X(S)$, where p(X=r) is the probability that X takes on value r
- Let S be the 36 outcomes (i, j) obtained by rolling 2 dice. The customary value of a roll is the random variable i+j. What is the distribution of this random variable?

Algorithmic Example

- Suppose a linear search algorithm is used to find the position of a given value in a list of length n containing the positive integers ≤ n in arbitrary order
- The number of comparisons performed is a random variable
  - What is its distribution?
  - What is its expected value?
- In general, expected value of a random variable X(s) is $E(X) = \sum_{s \in S} p(s)X(s)$