Discrete Mathematical Structures
CS 3233 Lecture 34

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November 30, 2005

Business

- Review session in lecture class room 2pm Monday, 12/5
- Final exam will be in regular lecture class room Friday 12/9 7:30-10:15am
- Dr. Winsborough will be out of town 12/7 – 12/9
  - No office hours Wed 12/7
  - Humayun will administer the final

Conditional Probability

- A fair coin is flipped 3 times. Assuming it comes up heads the first time, what is the probability it comes up tails an odd number of times?
- Definition
  - Let E and F be events with $p(F) > 0$. The conditional probability of E given F, denoted by $p(E|F)$, is $p(E|F) = p(E \cap F)/p(F)$
- Further examples
  - Suppose that a family has 2 children, one of which is a boy. What is the likelihood that the family has two boys?
  - Suppose that the first child is a boy. Now what is the likelihood that the family has two boys?

Independent Events

- Example
  - A fair coin is again flipped three times. Does knowing it comes up tails the first time (event F) alter the probability that it comes up tails an odd number of times (event E)?
- I.e., is $p(E|F) = p(E)$?
- Recall $p(E|F) = p(E \cap F)/p(F)$
- Definition
  - Events E and F are independent if and only if $p(E \cap F) = p(E)p(F)$
- Example
  - E = a family with 3 children has both sexes
    F = a family with 3 children has at most 1 boy
    Are E and F independent?

Random Variables

- Definition
  - A *random variable* X: S → R assigns a real number to each experiment outcome
  - The distribution of a random variable X is the set of pairs (r, p(X=r)) for r ∈ X(S), where p(X=r) is the probability that X takes on value r
- Example
  - Let S be the 36 outcomes (i, j) obtained by rolling 2 dice. The customary value of a roll is the random variable X(i,j) = i+j. What is the distribution of this random variable, assuming the dice are fair?

Algorithmic Example

- Suppose a linear search algorithm is used to find the position of a given value in a list of length n containing the positive integers ≤ n in arbitrary order
- The number of comparisons performed is a random variable
  - What is its distribution?
  - What is its expected value?
- Definition
  - In general, expected value of a random variable $X: S \rightarrow R$ is $E(X) = \sum_{s \in S} p(s)X(s)$