Discrete Mathematical Structures
CS 3233 Lecture 35

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Business

- Mock final and solutions are available on the course web page
  - It covers only material from November 7 onwards. Refer to midterms and mock midterms for prior material
  - The final is cumulative
- Bring your exams, mock exams, homework, etc., to review session
  - In lecture class room 2pm Monday, 12/5

Expected Value

- Suppose a fair coin is flipped 30 times. What is the expected (average) number of times it comes up heads?
- What is it if the coin is not fair, but comes up heads twice as often as it comes up tails?

Expected Value

- Definition
  - The expected value of a random variable $X: S \rightarrow R$ is $E(X) = \sum_{s \in S} p(s)X(s) = \sum_{r \in X(S)} p(X=r)r$
- Note that this is a weighted average
- Example
  - Suppose an urn contains 2 blue balls and 4 red balls, and that we select a ball at random 3 times, replacing the ball in the urn each time before we select again. What is the expected number of times we will select a blue ball?

Algorithmic Example

- Suppose a linear search algorithm is used to find the position of a given value in a list of length n containing the positive integers ≤ n in arbitrary order
- The number of comparisons performed is a random variable
  - What is its distribution?
  - What is its expected value?

Geometric Distribution

- Suppose the probability that a coin comes up tails is p. If this coin is flipped repeatedly until it comes up tails, what is the expected number of times it must be flipped?
  - S = {T, HT, HHT, HHHT, …}
  - $p(T) = p$, $p(HT) = (1-p)p$, $p(HHT) = (1-p)^2 p$ …
  - $E(X) = \Sigma\, j \cdot p(X=j) = \sum_{j=1}^{\infty} j \cdot (1-p)^{j-1} p = p \sum_{j=1}^{\infty} j \cdot (1-p)^{j-1}$
    $= p(1/p^2) = 1/p$
  - This uses $\sum_{j=1}^{\infty} j \cdot x^{j-1} = 1/(1-x)^2$ for $|x| < 1$, which follows from $\sum_{j=1}^{\infty} x^j = 1/(1-x)$ by differentiating each side