Discrete Mathematical Structures
CS 3233 Lecture Two

Prof. William Winsborough

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Assignment Due 9/2/05

- Section 1.1
  - 10, 16, 22, 24, 28e, 42, 46, 52, 54
- Section 1.2
  - 4, 8, 10, 26, 54

Implications Related to $p \rightarrow q$

- *Contrapositive*: $\neg q \rightarrow \neg p$
- *Converse*: $q \rightarrow p$
- *Inverse*: $\neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	T	T

Biconditionals

- Def: Given propositions p and q, $p \leftrightarrow q$ is a *biconditional*
  - $p \leftrightarrow q$ means “p if and only if q”
- Truth table for $p \leftrightarrow q$:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence

- Should $\neg q \rightarrow \neg p$ be interpreted as
  - $(\neg q) \rightarrow (\neg p)$, or as
  - $\neg(q \rightarrow (\neg p))$?
- Precedence gives rules for implicit parentheses

Op	Prec
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

A Note on Notation and Terminology Not in the Book

- Simple propositions are propositional variables, such as *p* and *q*, or one of the constants, **T** or **F**
- A *propositional formula* is either a simple proposition or a compound proposition
- Often Greek letters, such as ϕ and ψ, are used to denote propositional formulas, and Roman letters are reserved for propositional variables
- The book does not do this. It sometimes uses *p* and *q* to denote compound propositions, which seems potentially confusing

Translating English to Logic

- Fred can access the wireless network only if Fred has paid his tuition
  - Let a represent “Fred can access wireless”
  - Let t represent “Fred has paid his tuition”
  - $a \rightarrow t$

Consistency

- Intuitive requirement: specifications should not contain conflicting requirements
- Definition: A collection of propositional formulas is *consistent* if there is a truth assignment that makes all the expressions true

Example Inconsistent Spec

- Specification:
  - Fred can access the wireless network only if Fred has paid his tuition ($a \to t$)
  - Fred can access the wireless network (a)
  - Fred has not paid his tuition ($\neg t$)

a	t	$a \to t$	$\neg t$	$(a \to t) \wedge a \wedge \neg t$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

- $(a \to t) \wedge a \wedge \neg t$ is a *contradiction*

A More Concise Truth Table

$((a$	$\rightarrow$	$t)$	$\wedge$	$a)$	$\wedge$	$\neg$	t
T		T		T			T
T		F		T			F
F		T		F			T
F		F		F			F

A More Concise Truth Table

$((a$	$\rightarrow$	$t)$	$\wedge$	$a)$	$\wedge$	$\neg$	t
T	T	T		T		F	T
T	F	F		T		T	F
F	T	T		F		F	T
F	T	F		F		T	F

A More Concise Truth Table

$((a$	$\rightarrow$	$t)$	$\wedge$	$a)$	$\wedge$	$\neg$	t
T	T	T	T	T		F	T
T	F	F	F	T		T	F
F	T	T	F	F		F	T
F	T	F	F	F		T	F

A More Concise Truth Table

$((a$	$\rightarrow$	$t)$	$\wedge$	$a)$	$\wedge$	$\neg$	t
T	T	T	T	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	F	F	F	F	T
F	T	F	F	F	F	T	F

Bitwise Operations

1100	
1010	
1110	Bitwise OR
1000	Bitwise AND
0110	Bitwise XOR

Tautologies and Contradictions

- A compound proposition that is true for all truth assignments is a *tautology*
  - *E.g.*, $(p \to q) \leftrightarrow (\neg q \to \neg p)$
- One that is false for all assignments is a *contradiction*
- Given propositional formulas *p* and *q*, if the biconditional $p \leftrightarrow q$ is a tautology, then *p* and *q* are *logically equivalent*
  - In this case we write $p \equiv q$
    - e.g., $(p \to q) \equiv (\neg q \to \neg p)$
  - Note that ≡ is not a logical connective (*i.e.,* a logical operator), so $p \equiv q$ is not a compound proposition
  - ≡ is part of the *meta-language* we use for discussing formulas in the *object language* of propositional calculus

Some Important Logical Equivalences

$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotency laws
$\neg(\neg p) \equiv p$	Double nagation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commuitivity laws

More Equivalences

$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv q \land (p \land r)$	Associativity laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributivity laws
$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$	De Morgan's laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption laws
$p \lor \neg p \equiv T$ $p \land \neg p \equiv F$	Negation laws