Discrete Mathematical Structures

CS 3233 Lecture Three

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Proving Equivalences By Using Truth Tables

- Example 3 in Section 1.2 of text:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg q$	$p \vee \neg q$
T	T	F	T
T	F	T	F
F	T	F	T
F	F	T	T

Some Important Logical Equivalences (with Greek)

$\phi \wedge T \equiv \phi$ $\phi \vee F \equiv \phi$	Identity laws
$\phi \vee T \equiv T$ $\phi \wedge F \equiv F$	Domination laws
$\phi \vee \phi \equiv \phi$ $\phi \wedge \phi \equiv \phi$	Idempotency laws
$\neg(\neg\phi) \equiv \phi$	Double nagation law
$\phi_1 \vee \phi_2 \equiv \phi_2 \vee \phi_1$ $\phi_1 \wedge \phi_2 \equiv \phi_2 \wedge \phi_1$	Commuitivity laws

More Equivalences

$(\phi_1 \vee \phi_2) \vee \phi_3 \equiv \phi_1 \vee (\phi_2 \vee \phi_3)$ $(\phi_1 \wedge \phi_2) \wedge \phi_3 \equiv \phi_2 \wedge (\phi_1 \wedge \phi_3)$	Associativity laws
$\phi_1 \vee (\phi_2 \wedge \phi_3) \equiv (\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$ $\phi_1 \wedge (\phi_2 \vee \phi_3) \equiv (\phi_1 \wedge \phi_2) \vee (\phi_1 \wedge \phi_3)$	Distributivity laws
$\neg(\phi_1 \wedge \phi_2) \equiv \neg\phi_1 \vee \neg\phi_2$ $\neg(\phi_1 \vee \phi_2) \equiv \neg\phi_1 \wedge \neg\phi_2$	De Morgan’s laws
$\phi_1 \vee (\phi_1 \wedge \phi_2) \equiv \phi_1$ $\phi_1 \wedge (\phi_1 \vee \phi_2) \equiv \phi_1$	Absorption laws
$\phi_1 \vee \neg\phi_1 \equiv T$ $\phi_1 \wedge \neg\phi_1 \equiv F$	Negation laws

Proofs Based on Known Equivalences

- See Example 5 in Section 1.2 of text
- Note that when the text uses $\neg p \wedge p \equiv F$ this is really two steps, one using commutativity and one using the second negation law