Discrete Mathematical Structures

CS 3233 Lecture Four

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Predicates

- Propositional functions
- Examples
  - $p(x) \equiv x > 3$
  - $q(y) \equiv$ y has paid his tuition
  - $r(z) \equiv$ z has wireless access on campus
- $p(4)$ has a truth value
- What does $r(x) \rightarrow q(x)$ mean?

Predicates, Continued

- Each propositional function takes a fixed number of arguments
  – older(x,y) ≡ x is older than y
  – Here “older” is being used as a propositional function
- A propositional function p and a predicate p are the same thing
- A statement of the form $p(x_1, x_2, x_n)$ is a proposition

Formulas Involving Predicates

- Let $\phi(y) \equiv q(y) \to r(y)$
  - We say ϕ is *a formula in* y (not in text)
- Then $\phi(\text{Fred}) \equiv q(\text{Fred}) \to r(\text{Fred})$
  - This is an example of substitution of variables by values

Universe of Discourse and Quantifiers

- The *universe of discourse* or *domain* is the set of all possible values for variables
- We can refer to values in the universe either by using constant symbols (like “Fred”) or by using quantifiers
- There are two quantifiers in standard predicate calculus: *for all* ($\forall$) and *there exists* ($\exists$)
- There are called the universal quantifier and the existential quantifier, respectively

Universal Quantifiers

- The universal quantification of $p(x)$ is the proposition
  - "$p(x)$ is true for all values of x in the universe of discourse"
  - Written $\forall x\ p(x)$ or $\forall x.p(x)$
- Similarly, if $\phi(x)$ is a formula in x, $\forall x.\phi(x)$ means the formula holds for all elements of the universe
  - What does $\forall x.(r(x) \rightarrow q(x))$ mean?

Universal Quantifiers

- Note that $\forall x.p(x) \equiv \forall y.p(y)$
- If the universe of discourse is {0, 1, 2}, then $\forall x.p(x) \equiv p(0) \land p(1) \land p(2)$
- Can you always rewrite $\forall x.p(x)$ this way?
  - What if the universe of discourse is infinite?
  - Logical statements are finite objects

Existential Quantifiers

- The existential quantification of p(x) is the proposition
  - "there exists an element x in the universe of discourse such that p(x) is true"
  - Written $\exists x\ p(x)$ or $\exists x.p(x)$
  - What does $\exists x.(r(x) \land \neg q(x))$ mean?
- If the universe of discourse is {0,1,2}, then
  - $\exists x.p(x) \equiv p(0) \lor p(1) \lor p(2)$

Negations of Quantified Formulas

- $\neg \exists x.p(x) \equiv \forall x.\neg p(x)$
- $\neg \forall x.p(x) \equiv \exists x.\neg p(x)$