Discrete Mathematical Structures
CS 3233 Lecture Five

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Outline of Lecture

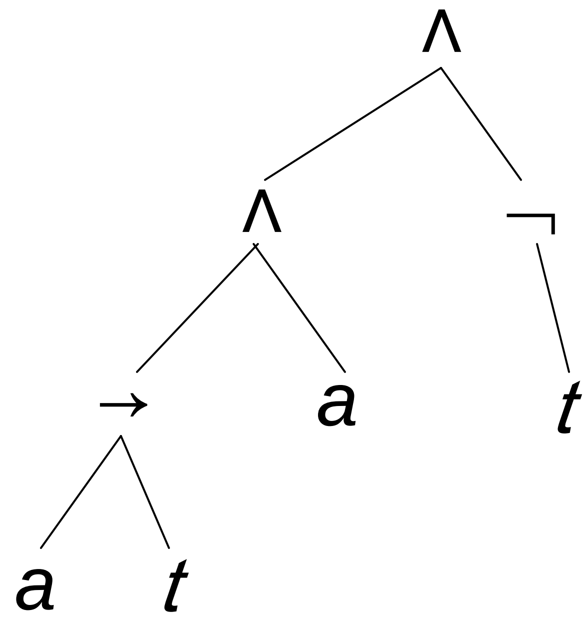
- Propositional logic
  - Precise recursive definition of set of propositional formulas
  - Expression trees and the concise truth table representation
- Predicate logic
  - Combining quantifiers

Recursive Definition of Propositional Formulas (Not in Text)

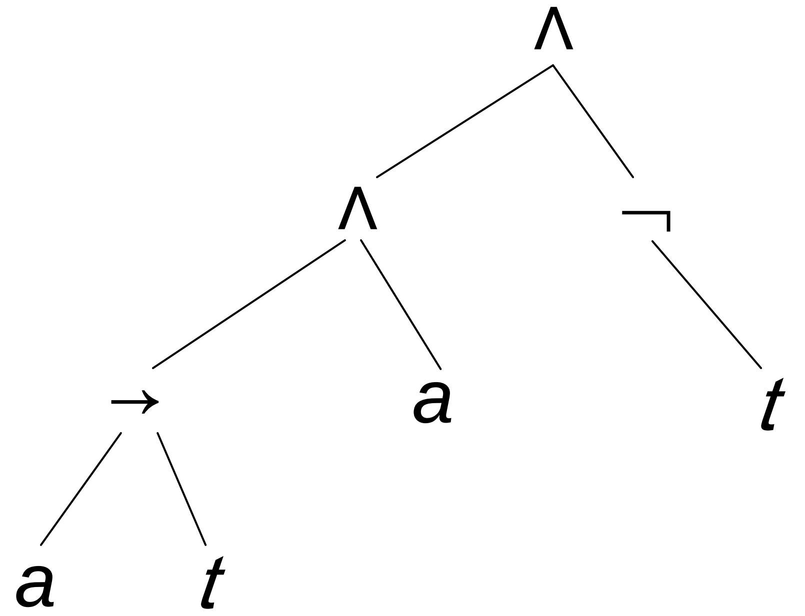
- Definition of *propositional formula:*
  - A propositional variable *p* is a propositional formula
  - The constants **T** and **F** are propositional formulas
  - If ϕ and ψ are propositional formulas, then the following are also propositional formulas:
    - (ϕ)
    - $\neg\phi$
    - $\phi \wedge \psi$
    - $\phi \vee \psi$
    - $\phi \rightarrow \psi$
    - $\phi \leftrightarrow \psi$
    - $\phi \oplus \psi$

Expression Trees (Not in Text)

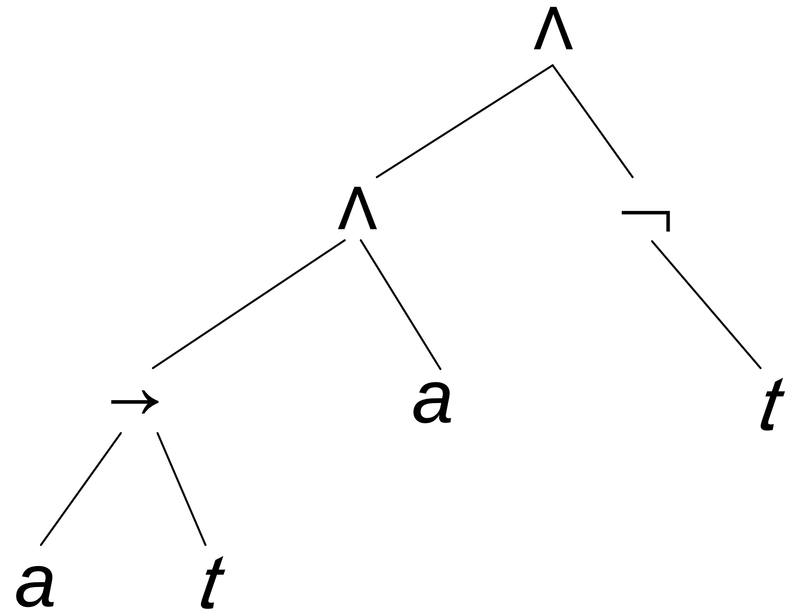
- Example: $((a \to t) \wedge a) \wedge \neg t$



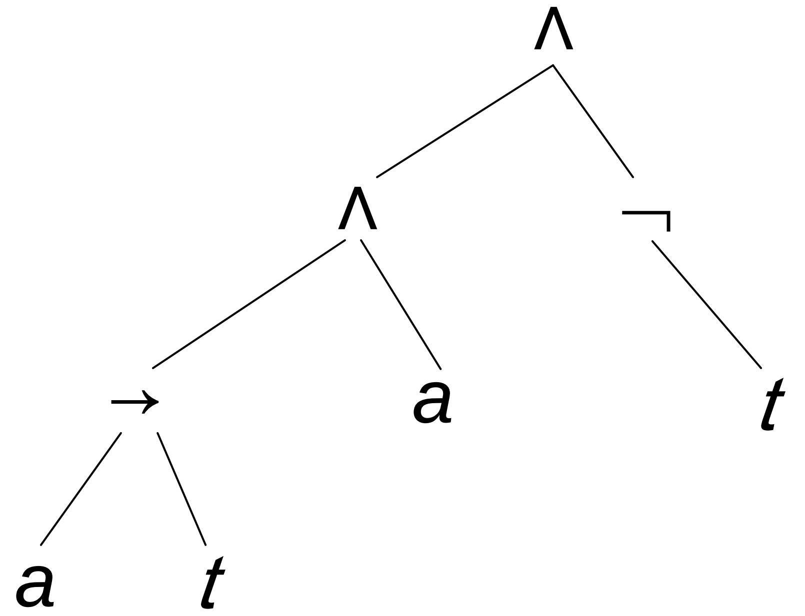
- Construct compact truth table by using expression tree



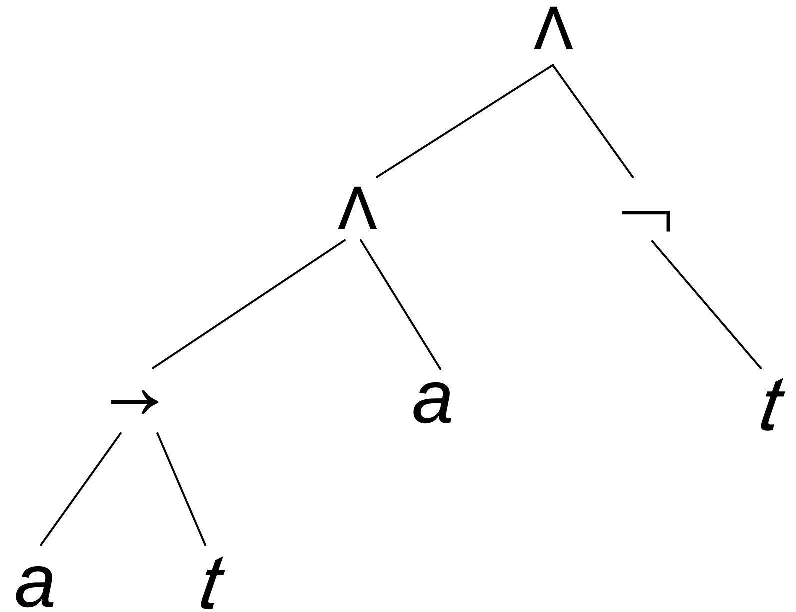
((*a*	→	*t*)	∧	*a*)	∧	¬	*t*
T		T		T			T
T		F		T			F
F		T		F			T
F		F		F			F



((*a*	→	*t*)	∧	*a*)	∧	¬	*t*
T	T	T		T		F	T
T	F	F		T		T	F
F	T	T		F		F	T
F	T	F		F		T	F



((*a*	→	*t*)	∧	*a*)	∧	¬	*t*
T	T	T	T	T		F	T
T	F	F	F	T		T	F
F	T	T	F	F		F	T
F	T	F	F	F		T	F



((*a*	→	*t*)	∧	*a*)	∧	¬	*t*
T	T	T	T	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	F	F	F	F	T
F	T	F	F	F	F	T	F

Truth Assignments

- Not formalized in text
- A *truth assignment* σ is a function that maps propositional variables to {true, false}

Recursive Definition of Satisfaction

- Not formalize in text
- A truth assignment σ *satisfies* a propositional formula ϕ (written $\sigma \models \phi$) if and only if:
  - ϕ is a propositional variable P and σ(P) = true
  - ϕ has the form $\neg\psi$ and σ does not satisfy ψ
  - ϕ has the form $\psi_1 \wedge \psi_2$ and $\sigma \models \psi_1$ and $\sigma \models \psi_2$
  - ϕ has the form $\psi_1 \vee \psi_2$ and $\sigma \models \psi_1$ or $\sigma \models \psi_2$
  - *Etc*.

Back to Quantifiers

- For the domain of integers, is $\forall x.(x>3)$ true or false?
  - How about $\exists x.(x>3)$?
  - Note: ">" is the predicate symbol here
  - X>3 is another way of writing >(x,3)
- What does $\forall x\ \forall y.(x+y = y+x)$ mean?
  - Is it true?
  - What is the predicate here?
  - In logic, "+" is called a *function symbol* (term not introduced in the text)

Scope of Quantifiers

- $(\exists x.x>3) \wedge (\exists x.x<1)$
- The scope of a quantifier is the formula following the dot

Open and Closed Formulas

- Not in text
- A formula is *closed* if all variables are in the scope of some quantifier
- Otherwise, the formula is *open*
- $p(x) \equiv \forall y. x \leq y$ is open