Discrete Mathematical Structures

CS 3233 Lecture Nine

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New Assignment & Questions

- Due September 21
- Section 1.6
  - 2, 8, 10, 12, 14, 16, 18, 22, 26, 30
- Section 1.7
  - 2, 4, 6, 8, 12, 16, 24, 26, 38
- How is the new problem set going?
- Are you going to come to my office hours?

Revisiting Example Existence Proof

- Constructive example: Sec.1.5, prob.49,p.76
  - "Theorem there exists a sequence of 100 consecutive positive integers that are not perfect squares
  - Claim: There are no perfect squares in the range (100^2,…,101^2)

Proof by Contradiction

- Suppose for contradiction that there exists $x \in \mathbf{Z}^+$ such that $100^2 < x^2 < 101^2$.
  - Clearly x is neither 100 nor 101
  - Case 1: $x < 100$. In this case there exists $d \in \mathbf{Z}^+$ such that $x+d = 100$. It follows that $(x+d)^2 = 100^2$. But $(x+d)^2 = x^2 + 2xd + d^2$. Since $x,d \in \mathbf{Z}^+$, it follows that $x^2 = 100^2 - (2xd + d^2) < 100^2$ contradicting our assumption in this case.
  - Case 2: $x > 101$. A similar contradiction is easily obtained in this case as well.
  - This concludes the proof

Clarification of a Definition

- The *set builder* notation constructs a set by stating the property or properties elements must have to be in the set.
  - E,g., {x|x is a prime number less than 100}

Relationships Between Sets

- Two sets A and B are *equal* if A and B have the same elements
- A is a *subset* of B if every element of A is also an element of B
  - Written: $A \subseteq B$
  - $\forall x (x \in A \rightarrow x \in B)$
  - If $A \subseteq B$ and $A \neq B$, then A is a *proper subset* of B, written: $A \subset B$
- Proposition (obvious theorem):
  - $\emptyset \subseteq A$ and $A \subseteq A$

Cardinality

- Def: Let S be a set. If S contains exactly *n* distinct elements, for some nonnegative integer *n*, we say S is a *finite set* and that *n* is the cardinality of S.
  - Denoted |S| = *n*
- Otherwise, S is said to be *infinite*

Power Set

- Def: Given a set S, the *power set* of S is the set of all subsets of S
  - Denoted ***P***(S)
  - ***P***(S) = { A | A ⊆ S}
- Examples
  - P({1,2}) = {∅, {1}, {2}, {1,2}}
  - P(∅) = {∅}
  - P({∅}) = {∅,{∅}}

Cartesian Products

- Def: An *n-tuple* $(a_1, a_2, \ldots, a_n)$ is an ordered collection whose elements are indexed by the integers 1, 2, …,n
- Def: Given sets A and B, the *Cartesian product* of A and B, denoted A x B, is the set of ordered pairs:
  - $A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$
- Can generalize to n-tuples:
  - $A_1 \times A_2 \times \ldots \times A_n$
    $= \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i \in 1,\ldots,n\}\}$

Unions and Intersections

- Given sets A and B, the *union* of A and B, A∪B, is the set containing each element of A and each element of B
- Given sets A and B, the *intersection* of A and B, A ∩ B, is the set consisting of objects that are elements of both A and B
- Venn diagrams help visualize

Disjoint Sets, Set Difference

- Sets A and B are *disjoint* if A ∩ B = ∅
- The *difference* of A and B, A – B, consists of all elements of A that are not elements of B
  - Also called the *complement of* B *with respect to* A
- Given universe U and set S, the *compliment* of S, is the compliment of S with respect to U

Identities and Big Unions, Intersections

- Set Identities
  - Review Table 1
  - Review Example 11
- Given a collection of sets $A_1, A_2, \cdots, A_n$
  - $\bigcup_{1\le i\le n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$
  - $\bigcap_{1\le i\le n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$