Discrete Mathematical Structures
CS 3233 Lecture Ten

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Checking In

- The 10% penalty for late assignments will be waived this week.
  - Assignment 2 may not be turned in after 2pm Thursday 9/15
- Any questions?
  - General
  - Last lecture
  - Assignments
  - Schedule

Set Identities

- Given sets A and B,
  A = B if and only if $A \subseteq B$ and $B \subseteq A$
- Set Identities
  - Review Table 1, p.89
    - Use Venn diagrams to illustrate distributivity and De Morgan's laws
  - Review Example 11

Introduction to Functions

- Section 1.8
- Definition
  - Given sets A and B, a *function from* A *to* B is an assignment of exactly one element of B to each element of A
  - For $a \in A$, we write $f(a) = b$ if $b \in B$ is the unique element associated with a by f
  - We write $f : A \rightarrow B$ to indicate that f is a function from A to B
    - $A \rightarrow B$ is called the *type* of f

Function Graphs

- This is a different usage of “graph” than:
  - The plot of f(x) on the Cartesian plane
  - A set of nodes and a set of edges
- Definition
  - Given a function f : A → B, the *graph* of f is the set of ordered pairs {(a,b) | a ∈ A and b = f(a)}
- Formally speaking, a function <u>is</u> its graph
  - Thus, f is a subset of A x B in which each element of A occurs exactly once

Terminology

- Let $f : A \rightarrow B$
  - We say f *maps* A to B
  - A is the *domain* of f
  - B is the *codomain* of f
  - If f(a) = b,
    - b is the *image* of a and
    - a is the *pre-image* of b
  - If $S \subseteq A$, the *image* of S is $\{f(s) \mid s \in S\}$ and is denoted by f(S)
  - $f(A) = \{f(a) \mid a \in A\}$ is called the *range* of f

Examples

- Successor function
  - $s : N \rightarrow N$
  - $s(n) = n+1$
- Floor
  - $floor : R \rightarrow Z$
  - floor(x) = greatest integer less than x
- Square root
  - $\sqrt{} : R \rightarrow C$
  - The square root of a real is a complex number
- Truth assignment
  - $\sigma : \mathcal{V} \rightarrow$ {true, false}

Injections and Surjections

- f : A → B is *one-to-one*, or *injective*, if and only if f(x) = f(y) implies x = y for all x, y ∈ A
  - ∀x∀y(f(x) = f(y) → x = y)
  - Equivalently, ∀x∀y(x ≠ y → f(x) ≠ f(y))
  - f is called an *injection*
- f : A → B is *onto,* or *surjective*, if for every b ∈ B, there is an a ∈ A such that f(a) = b
  - f is *a function of* A *onto* B
  - ∀y∃x(f(x) = y)
  - f is called a *surjection*

Bijections

- If f : A → B is injective and surjective, it is *bijective*

Examples

- Successor
- Floor
- Square root
- Truth assignment
- Mapping of 32-bit words to Z
- Successor in $Z_3 = \{0, 1, 2\}$
  - s(0) = 1
  - s(1) = 2
  - s(2) = 0