Discrete Mathematical Structures
CS 3233 Lecture Seven

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Anonymous Feedback

- Thanks!
- Some people wanted me to move faster
- Some wanted me to work more problems like the homework
  - I may be able to do a little of that, but this need should be addressed primarily in recitation
  - And in office hours

Feedback on Assignment 1

- I was surprised how many people didn’t answer all the problems
  – The problems were designed to test you basic understanding
- I was *very* surprised that I had almost no one in my office for office hours
  – Everyone should come to office hours from time to time

Homework Assignment 2
Due Wed. 9/14/05

- Section 1.3
  - 6(c) -- (f), 8, 10, 34, 42, 46
  - In problem 8, also say whether you think the statement is true and why
  - Hint: problem 43 is similar to 42 and is solved in the back of the book
- Section 1.4
  - Problems 6, 8, 10, 44, 46
  - (problems 9 and 45 are similar to 8 and 44, and are solved in the back of the book)
- Section 1.5
  - 2, 6, 10, 12, 20, 34, 48

Scheduling Second Make-up Lecture

- There will be a make-up lecture presented during recitation September 19 and 21
  - Mark your calendar
- The same lecture will be given both days
- This is in addition to the regular lectures at the regular times those days

Rules of Inference for Universally Quantified Statements

- Universal instantiation

$$\frac{\forall x.\ p(x)}{p(c)}$$

  – For any c

- Universal generalization

$$\frac{p(c)}{\forall x.\ p(x)}$$

  – Must show p(c) for arbitrary c

Rules of Inference for Existentially Quantified Statements

- Existential instantiation

$$\frac{\exists x.\ p(x)}{p(c)}$$

  - c must be a new name (constant) that does not appear earlier in the proof
- Existential generalization

$$\frac{p(c)}{\exists x.\ p(x)}$$

  - c can be any name

Using Rules of Inference for Quantified Statements

- Example 13, p.62
- We won't always be so meticulous about the application of rules of inference
  - But it's important to understand the fundamentals

Some Important Methods of Proof

- Direct proof of $p \rightarrow q$
  - Assume p, derive q
- Indirect proof: directly prove the contrapositive
  - Assume $\neg q$, derive $\neg p$
- Vacuous and trivial proofs
  - Show that $p \rightarrow q$ holds by showing that p does not hold
- Proof by contradiction
  - Prove p by directly proving $\neg p \rightarrow F$

Example Proof by Contradiction

- Example 21, p.66:
  - Def: a real number *r* is *rational* if there exist integers *s* and *t* such that $r = s / t$. Otherwise *r* is called *irrational*
  - Theorem: $\sqrt{2}$ is irrational
  - Proof Strategy: Assume $\sqrt{2}$ is rational and derive a contradiction (a false statement)
    - p is "$\sqrt{2}$ is irrational"
    - Directly prove $\neg p \rightarrow F$

More Important Methods of Proof

- Proof by case analysis
- Proof of equivalence
- Existence proofs
  - Constructive versus nonconstructive
  - Constructive example: Sec.1.5, prob.49,p.76
    - "Prove there are 100 consecutive positive integers that are not perfect squares"
    - Proof sketch: consider the integers between 100^2 and 101^2
  - Nonconstructive example: Example 27,p.69