Discrete Mathematical Structures
CS 3233 Lecture Eight

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Any Questions

- Were you able to understand the details of the proof by contradiction we outlined last time?
- Were you able to understand how the TA graded your assignment?
  - If he made any mistakes, did you point them out to him and get them corrected?
- Have you started the new assignment?

More Important Methods of Proof

- Proof by case analysis
- Proof of equivalence
- Existence proofs
  - Constructive versus nonconstructive
  - Constructive example: Sec.1.5, prob.49,p.76
    - “Prove there are 100 consecutive positive integers that are not perfect squares”
    - Proof sketch: consider the integers between 100^2 and 101^2
  - Nonconstructive example: Example 27,p.69
    - Theorem: There exist irrational numbers *x* and *y* such that x^y is rational

Beginning of Section 1.6, 1.7

- Sets and set operations
- Def: a *set* is an unordered collection of objects
  - We will work with “naïve set theory,” in which the objects can be anything.
  - This can lead to logical inconsistencies called “paradoxes,” which we will touch on later

Terminology

- Def: Objects in a set are called its *elements* or *members*. A set is said to *contain* its members
- Set notations:
  - $\varnothing = \{\ \}$ the *empty set*
  - {a, e, i, o, u}
  - {1, 2, 3, …, 100}
  - $\mathbf{N} = \{0, 1, 2, \ldots\}$
  - $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ the *integers*
  - $\mathbf{Q} = \{p/q \mid p \in \mathrm{Z},\ q \in \mathrm{Z},\ q \neq 0\}$
- The first and last of these examples are more formal than the others because they are more precise

Relationships Between Sets

- Two sets A and B are *equal* if A and B have the same elements
- A is a *subset* of B if every element of A is also an element of B
  - Written: $A \subseteq B$
  - $\forall x\ (x \in A \rightarrow x \in B)$
  - If $A \subseteq B$ and $A \neq B$, then A is a *proper subset* of B, written: $A \subset B$
- Proposition (obvious theorem):
  - $\emptyset \subseteq A$ and $A \subseteq A$