Mock Exam for Midterm II
Discrete Mathematical Structures CS3233

November, 2005

1. Define $f(n) = \mathcal{O}(g(n))$, $f(n) = \Omega(g(n))$, and $f(n) = \Theta(g(n))$.

2. Prove or disprove: $(n^2 + n^3)/2 = \Theta(n^3)$.

3. What is the best big-O estimate of the number of comparisons that are performed by an algorithm that takes a list of n integers and finds the least of the first 100 values? Justify your answer.

4. What is the expected-case complexity of finding the least value in a list of n integers? Select the one best answer from the following list: $\mathcal{O}(1), \mathcal{O}(\log n), \mathcal{O}(n), \mathcal{O}(n \log n), \mathcal{O}(n^2), \mathcal{O}(n^3), \mathcal{O}(2^n)$?

5. How is the complexity of a problem defined?

6. What are P and NP?

7. Can an NP-Complete problem be solved by a (deterministic) algorithm having polynomial complexity?

8. Suppose the integer m ends with 10 zeros. What can you conclude about m's prime factorization?

9. Let a and b be positive integers, let $p_1, p_2, \ldots, p_n$ enumerate the primes less than $max(a, b)$ in increasing order, and let $a = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n}$ and $b = p_1^{b_1} p_2^{b_2} \cdots p_n^{b_n}$ be the prime factorizations of a and b.

   Prove that $\text{lcm}(a, b) = p_1^{\max(a_1,b_1)} p_2^{\max(a_2,b_2)} \cdots p_n^{\max(a_n,b_n)}$ by showing: (a) That both a and b divide $p_1^{\max(a_1,b_1)} p_2^{\max(a_2,b_2)} \cdots p_n^{\max(a_n,b_n)}$, and by showing (b) That no smaller number is divisible by both a and b.

10. Given positive integers a and b, prove that $\gcd(a, b) \cdot \text{lcm}(a, b) = ab$.

11. Let m be a positive integer. Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a + c \equiv b + d \pmod{m}$ and $ac \equiv bd \pmod{m}$.

12. Use induction to show that $P(n) \equiv \sum_{1 \le i \le n} (-1)^{i-1} i^2 = (-1)^{n-1} n(n + 1)/2$ holds for all positive integers n.

13. Is the set of positive rational numbers well ordered? Why or why not?

14. Is the set of negative integers well ordered? Why or why not?

15. Is the set of integers greater than 100 well ordered? Why or why not?