

Discrete Mathematical Structures
CS 3233 Lecture 14

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Business

- Read section 3.2 for Thursday
- Homework 5 will be due
Thursday 19 October
- Homework 6
Due Tuesday 24 October
– 3.2: 6, 8, 10, 14, 20, 22

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Bubble Sort

```
proc bubble sort( $a_1, a_2, \dots, a_n$ )  
for  $i := 1$  to  $n-1$   
  for  $j := 1$  to  $n-i$   
    if  $a_j > a_{j+1}$  then interchange  $a_j$  and  $a_{j+1}$   
  { $a_1, a_2, \dots, a_n$  is in increasing order}
```

- How many comparisons are performed?

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Insertion Sort

```
proc insertion sort( $a_1, a_2, \dots, a_n$ : reals with  $n \geq 2$ )  
for  $j := 2$  to  $n$   
  begin  
     $i := 1$   
    while  $a_j > a_i$   
       $i := i+1$   
     $m := a_j$   
    for  $k := 0$  to  $j - i - 1$   
       $a_{j-k} := a_{j-k-1}$   
     $a_i := m$   
  end { $a_1, a_2, \dots, a_n$  is in increasing order}
```

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Growth of Functions

- Big-O Notation
- Def: Let $f, g : Z \rightarrow R$. We say $f(x)$ is $O(g(x))$ if there are constants C and k such that $|f(x)| \leq C|g(x)|$ for all $x > k$
 - We say “ $f(x)$ is big-oh of $g(x)$ ”
- When discussing positive-valued functions, we can drop the $|\cdot|$

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Example

- Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2$
- $f(x)$ is $O(g(x))$
 - $k = 1$ and $C = 4$ (witnesses)
 - $k = 2$ and $C = 3$
- $g(x)$ is $O(f(x))$
 - $k = 1$ and $C = 1$

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Example

- Let $f(x) = 7x^2$ and $h(x) = x^3$
- $7x^2$ is $O(x^3)$
 - $k = 1$ and $C = 7$
- x^3 is not $O(7x^2)$
 - There is no C such that $x^3 \leq C(7x^2)$ for “large” x

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Degree of Polynomial is What Matters

- Theorem:
Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$.
Then $f(x)$ is $O(x^n)$

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More Examples

- Sum of first n positive integers

$$1+2+\dots+n \leq \underbrace{n+n+\dots+n}_n = n^2$$

So $\sum_{i=1}^n i$ is $O(n^2)$

- It follows that the complexity of both bubble sort and insertion sort is $O(n^2)$