Discrete Mathematical Structures
CS 3233 Lecture 19

Prof. William Winsborough
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Business

- Read sections 5.1 and 5.2
- Homework due Tuesday 21 November
  - 5.1: 8, 12, 14, 16, 24
  - 5.2: 2, 4, 8
- Return exam today and discuss grades

Sorted w/o HW ↓ | **Current Grade Standing** | Sorted w/ HW ↓

Avg No HW	Avg w/HW		Avg No HW	Avg w/HW	
95.5	82.3		94.5	93.1	
94.5	93.1		88.5	87.9	
92.5	83.5		84.0	84.7	
88.5	87.9	A	92.5	83.5	
84.0	84.7		83.0	83.3	
83.0	83.3		95.5	82.3	
81.5	82.2		81.5	82.2	
80.0	80.8		80.0	80.8	
78.5	77.5		78.5	77.5	
78.0	69.4		77.0	77.1	
77.0	77.1		69.0	73.5	
75.5	65.3	B	68.0	70.9	
72.0	69.3		78.0	69.4	
71.5	64.1		72.0	69.3	
69.0	73.5		64.0	67.8	Earns a B
68.5	66.6		68.5	66.6	
68.0	70.9		75.5	65.3	
65.0	57.1		71.5	64.1	
64.0	67.8		57.0	62.5	
63.5	51.1	C	65.0	57.1	
57.0	62.5		52.5	51.4	
52.5	51.4		63.5	51.1	
44.5	35.2		36.5	44.8	
38.5	40.9	D	38.5	40.9	
36.5	44.8		44.5	35.2	

Counting

- We are beginning Chapter 5 today
- Basic question: how can we determine how many configurations or procedures are possible in a given system?

Product Rule

- Suppose a system or a procedure can be broken down in to two components. If there are n_1 possible ways of configuring the first component, and n_2 ways of configuring the second, then the overall system has $n_1 n_2$ configurations

Examples

- 32 computers with 24 ports each have a total of 768 ports
- Number of bit strings of length 7 is 128
- License plates
- One to one functions
- Nested for loops
- Subsets of a finite set

The Sum Rule

- Given two disjoint sets, one of size n_1, the other of size n_2, the number of ways to choose one element from one of the two sets is n_1+n_2

Examples

- How many ways are there to choose a character if upper and lowercase alphabetics and numbers are all allowed?

Counting Example

- How many passwords if they must be 8 alphanumeric characters long and must include at least one numeric?
  - Idea: all 8 character strings minus the ones that contain no numerics

Principle of Inclusion-Exclusion

- $|A \cup B| = |A| + |B| - |A \cap B|$
- Example
  - How many bit strings of length 8 start with a 1 or end with 00?

Pigeonhole Principle

- If k+1 or more objects are placed in k boxes, there is at least one box containing two or more objects
- Proof
  - If each box contains at most 1 object, together they can contain at most k objects

Generalized Pigeonhole Principle

- If N objects are placed into k boxes, at least one box contains at least $\lceil N/k \rceil$
- Proof: Suppose each box contains at most $\lceil N/k \rceil - 1$ objects. This doesn't account for all the objects:

  $k(\lceil N/k \rceil - 1) < k((N/k + 1) - 1) = N$
- Example
  - Among 100 people there are at least $\lceil 100/12 \rceil = 9$ people born in the same month