Discrete Mathematical Structures
CS 3233 Lecture 20

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Business

- Read section 5.3
- Recall: Homework due Tuesday 21 Nov.
  - 5.1: 8, 12, 14, 16, 24
  - 5.2: 2, 4, 8
- Student evaluations will be done today

Counting Examples

- How many strings of lowercase letters have length 4 or less?
- How many strings of ascii characters contain "@" at least once?
- How many strings of 3 decimal digits
  - Do not contain the same digit 3 times?
  - Begin with an even digit?
  - Have exactly two digits that are 4s?

Principle of Inclusion-Exclusion

- $|A \cup B| = |A| + |B| - |A \cap B|$
- Example
  - How many bit strings of length 8 start with a 1 or end with 00?

Pigeonhole Principle

- If k+1 or more objects are placed in k boxes, there is at least one box containing two or more objects
- Proof
  - If each box contains at most 1 object, together they can contain at most k objects

Generalized Pigeonhole Principle

- If N objects are placed into k boxes, at least one box contains at least $\lceil N/k \rceil$
- Proof: Suppose each box contains at most $\lceil N/k \rceil$ - 1 objects. This doesn't account for all the objects:

  $k(\lceil N/k \rceil - 1) < k((N/k + 1) - 1) = N$
- Example
  - Among 100 people there are at least $\lceil 100/12 \rceil = 9$ people born in the same month

Pigeonhole Examples

- Drawer contains 12 brown and 12 black socks
  - How many must be selected at random to ensure having two socks of the same color?
  - How many must be selected at random to ensure having two black socks?
- Use the pigeonhole principle to show that if $|A| < |B|$, then no $f: A \to B$ is surjective

Permuations

- Begin Section 4.3
- Definition
  - Given a set S, an *r-permutation of S* is an ordered arrangement of r distinct elements of S
- Theorem
  - Given a set S of size n, the number of r-permutations of S is $P(n,r) = n(n-1)(n-2)\ldots(n-r+1) = n!/(n-r)!$

Examples

- The number of alphabetic strings of length 3 consisting of distinct characters
- The number of one-to-one functions from a set of size r to a set of size n

Combinations

- Definition
  - Given a set S, an *r-combination of S* is an **unordered** arrangement of r distinct elements of S

Examples

- The number of sets of size three consisting of (distinct) alphabetic characters
- The number of subsets of size r drawn from a set of size n
  - Compared to the set of one-to-one functions from a set of size r to a set of size n, we are considering only the range of the function, not its individual values

A Way of Thinking

- How many ways are there to order a set of size n?
- If you only care about the first r places in the ordering, (n-r)! of the orderings are effectively the same
  - This is because once I've chosen the first r places, there remain (n-r) elements whose order I don't care about
  - Thus, the number of permutations is n!/(n-r)!
  - For the number of combinations, you also don't care about the ordering of the elements in the first r places, so you divide the number of permutations by the number or ways of ordering the first r places, which is r!