

Discrete Mathematical Structures  
CS 3233 Lecture 21

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## Business

- Read sections 6.1, 6.2
- Homework due Tuesday 28 Nov.
  - 5.2: 2, 4, 8 (These were due today, but are now moved into the next assignment)
  - 5.3: 12, 16, 24

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## Pigeonhole Principle

- If  $k+1$  or more objects are placed in  $k$  boxes, there is at least one box containing two or more objects
- Proof
  - If each box contains at most 1 object, together they can contain at most  $k$  objects

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## Generalized Pigeonhole Principle

- If  $N$  objects are placed into  $k$  boxes, at least one box contains at least  $\lceil N/k \rceil$
- Proof: Suppose each box contains at most  $\lceil N/k \rceil - 1$  objects. This doesn't account for all the objects:
$$k(\lceil N/k \rceil - 1) < k((N/k + 1) - 1) = N$$
- Example
  - Among 100 people there are at least  $\lceil 100/12 \rceil = 9$  people born in the same month

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## Pigeonhole Examples

- Drawer contains 12 brown and 12 black socks
  - How many must be selected at random to ensure having two socks of the same color?
  - How many must be selected at random to ensure having two black socks?
- Use the pigeonhole principle to show that if  $|A| > |B|$ , then no  $f:A \rightarrow B$  is injective

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## Permutations

- Definition
  - Given a set  $S$ , an  $r$ -permutation of  $S$  is an **ordered** arrangement of  $r$  distinct elements of  $S$
- Theorem
  - Given a set  $S$  of size  $n$ , the number of  $r$ -permutations of  $S$  is  $P(n,r) = n(n-1)(n-2)\dots(n-r+1) = n!/(n-r)!$

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## Examples

- The number of alphabetic strings of length 3 consisting of distinct characters
- The number of one-to-one functions from a set of size  $r$  to a set of size  $n$

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## Combinations

- Definition
  - Given a set  $S$ , an  $r$ -combination of  $S$  is an **unordered** arrangement of  $r$  distinct elements of  $S$
- Theorem
  - Given a set  $S$  of size  $n$ , and an integer  $r$ ,  $0 \leq r \leq n$ , the number of  $r$ -combinations of  $S$  is  $C(n,r) = n!/((n-r)!r!)$
- $C(n,r)$  is a *binomial coefficient*

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## Examples

- The number of sets of size three consisting of (distinct) alphabetic characters
- The number of subsets of size  $r$  drawn from a set of size  $n$ 
  - Compared to the set of one-to-one functions from a set of size  $r$  to a set of size  $n$ , we are considering only the range of the function, not its individual values

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## A Way of Thinking

- How many ways are there to order a set of size  $n$ ?
- If you only care about the first  $r$  places in the ordering,  $(n-r)!$  of the orderings are effectively the same
  - This is because once I've chosen the first  $r$  places, there remain  $(n-r)$  elements whose order I don't care about
  - Thus, the number of permutations is  $n!/(n-r)!$
  - For the number of combinations, you also don't care about the ordering of the elements in the first  $r$  places, so you divide the number of permutations by the number or ways of ordering the first  $r$  places, which is  $r!$

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