

Discrete Mathematical Structures
CS 3233 Lecture 22

Prof. William Winsborough
November 28, 2006

Business

- Recall: Read sections 6.1, 6.2
- Practice homework (will not be collected)
 - 6.1: 5, 7, 21, 37
 - 6.2: 1, 3, 5, 7, 9
- About half of next class period will be devoted to review

28 November 2006

Winsborough CS 3233 Lecture 22

2

Combinations

- Definition
 - Given a set S , an *r-combination* of S is an **unordered** arrangement of r distinct elements of S
- Theorem
 - Given a set S of size n , and an integer r , $0 \leq r \leq n$, the number of r -combinations of S is $C(n,r) = n!/((n-r)!r!)$
- $C(n,r)$ is a *binomial coefficient*

28 November 2006

Winsborough CS 3233 Lecture 22

3

Examples

- The number of sets of size three consisting of (distinct) alphabetic characters
- The number of subsets of size r drawn from a set of size n
 - Compared to the set of one-to-one functions from a set of size r to a set of size n , we are considering only the range of the function, not its individual values

28 November 2006

Winsborough CS 3233 Lecture 22

4

A Way of Thinking

- How many ways are there to order a set of size n ?
- If you only care about the first r places in the ordering, $(n-r)!$ of the orderings are effectively the same
 - This is because once I've chosen the first r places, there remain $(n-r)$ elements whose order I don't care about
 - Thus, the number of permutations is $n!/(n-r)!$
 - For the number of combinations, you also don't care about the ordering of the elements in the first r places, so you divide the number of permutations by the number of ways of ordering the first r places, which is $r!$

28 November 2006

Winsborough CS 3233 Lecture 22

5

Discrete Probability

- An *experiment* yields one of a given set of possible outcomes
- The *sample space* S is the set of possible outcomes
- An *event* E is a subset of the sample space
- Assuming each outcome in S is equally likely, the *probability* of event E is $p(E) = |E|/|S|$
 - This is just the sum of the probabilities of each outcome that belongs to the event

28 November 2006

Winsborough CS 3233 Lecture 22

6

Simple Probability Example

- An urn contains 4 blue balls and 5 red balls. If one ball is chosen at random, what is the probability that it is blue?
- Rolling two dice, what is the probability of rolling a 7?
- Lottery: what is the probability of correctly choosing a set of 6 positive integers ≤ 40 ?
- What is the probability of being dealt 4 of a kind in a 5-card poker hand?

28 November 2006

Winsborough CS 3233 Lecture 22

7

Combinations of Events

- Given an event $E \subseteq S$, \bar{E} is the complementary event, $S-E$.
 $p(\bar{E}) = 1 - p(E)$
 - If a coin is flipped 10 times, what is the probability of getting heads at least once?
- If E_1 and E_2 are events in S then what is $p(E_1 \cup E_2)$?
 - What's the probability that a positive integer ≤ 100 and selected at random is divisible by neither 2 nor 5?

28 November 2006

Winsborough CS 3233 Lecture 22

8

Probability Distributions

- What happens when all outcomes are not equally likely? Distribution is not *uniform*.
 - Given a finite sample space $S = \{x_1, \dots, x_n\}$, a real-valued function $p: S \rightarrow [0,1]$ is a *probability distribution* if
 1. $0 \leq p(x_i) \leq 1$, for $i = 1, 2, \dots, n$
 2. $\sum_{0 \leq i \leq n} p(x_i) = 1$
- Biased-coin example: heads twice as likely as tails
- Probability of event E : $p(E) = \sum_{x \in E} p(x)$

28 November 2006

Winsborough CS 3233 Lecture 22

9

Conditional Probability

- A fair coin is flipped 3 times. Assuming it comes up heads the first time, what is the probability it comes up tails an odd number of times?
- Definition
 - Let E and F be events with $p(F) > 0$. The conditional probability of E given F , denoted by $p(E|F)$, is $p(E|F) = p(E \cap F)/p(F)$
- Further examples
 - Suppose that a family has 2 children, one of which is a boy. What is the likelihood that the family has two boys?
 - Suppose that the first child is a boy. Now what is the likelihood that the family has two boys?

28 November 2006

Winsborough CS 3233 Lecture 22

10

Independent Events

- A fair coin is again flipped three times. Does knowing it comes up tails the first time alter the probability that it comes up tails an odd number of times?
- Definition
 - Events E and F are independent if and only if $p(E \cap F) = p(E)p(F)$
- E = a family with 3 children has both sexes
 F = a family with 3 children has at most 1 boy
Are E and F independent?

28 November 2006

Winsborough CS 3233 Lecture 22

11

Random Variables

- A *random variable* $X: S \rightarrow \mathbb{R}$ assigns a real number to each experiment outcome
- The distribution of a random variable X is the set of pairs $(r, p(X=r))$ for $r \in X(S)$, where $p(X=r)$ is the probability that X takes on value r
- Let S be the 36 outcomes (i, j) obtained by rolling 2 dice. The customary value of a roll is the random variable $i+j$. What is the distribution of this random variable?

28 November 2006

Winsborough CS 3233 Lecture 22

12

Algorithmic Example

- Suppose a linear search algorithm is used to find the position of a given value in a list of length n containing the positive integers $\leq n$ in arbitrary order
- The number of comparisons performed is a random variable
 - What is its distribution?
 - What is its expected value?
- In general, expected value of a random variable $X(s)$ is $E(X) = \sum_{s \in S} p(s)X(s)$