

Discrete Mathematical Structures

CS 3233 Lecture Two

Prof. William Winsborough

August 29, 2006

Business

- Questions???
- Read Sections 1.1, 1.2, and 1.3 by Thursday
- Homework due Tuesday September 5
 - Section 1.1: 6d, 6e, 6g, 10, 18a, 18c, 18e, 26, 28
 - Section 1.2: 2, 6, 10, 12

Translating English to Logic

- Winsborough will not finish his reviewing on time unless he hurries
 - Let h represent “Winsborough hurries”
 - Let f represent “Winsborough finishes on time”
- Is this statement true when Winsborough is already late?

Tautologies and Contradictions

- A compound proposition that is true for all truth assignments is called a *tautology*
 - E.g., $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- One that is false for all assignments is a *contradiction*
- Given propositional formulas ϕ and ψ , if the biconditional $\phi \leftrightarrow \psi$ is a tautology, then ϕ and ψ are *logically equivalent* (this is an alternate definition)
 - In this case we write $\phi \equiv \psi$
 - e.g., $(\phi \rightarrow \psi) \equiv (\neg \psi \rightarrow \neg \phi)$
 - Note that \equiv is not a logical connective (i.e., a logical operator), so $\phi \equiv \psi$ is not a (compound) proposition
 - \equiv is part of the *meta-language* we use for discussing formulas in the *object language* of propositional calculus

Some Important Logical Equivalences

$\phi \wedge T \equiv \phi$ $\phi \vee F \equiv \phi$	Identity laws
$\phi \vee T \equiv T$ $\phi \wedge F \equiv F$	Domination laws
$\phi \vee \phi \equiv \phi$ $\phi \wedge \phi \equiv \phi$	Idempotency laws
$\neg(\neg \phi) \equiv \phi$	Double negation law
$\phi \vee \psi \equiv \psi \vee \phi$ $\phi \wedge \psi \equiv \psi \wedge \phi$	Commutativity laws

More Equivalences

$(\phi \vee \psi) \vee \theta \equiv \phi \vee (\psi \vee \theta)$ $(\phi \wedge \psi) \wedge \theta \equiv \psi \wedge (\phi \wedge \theta)$	Associativity laws
$\phi \vee (\psi \wedge \theta) \equiv (\phi \vee \psi) \wedge (\phi \vee \theta)$ $\phi \wedge (\psi \vee \theta) \equiv (\phi \wedge \psi) \vee (\phi \wedge \theta)$	Distributivity laws
$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$ $\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	De Morgan's laws
$\phi \vee (\phi \wedge \psi) \equiv \phi$ $\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption laws
$\phi \vee \neg\phi \equiv \text{T}$ $\phi \wedge \neg\phi \equiv \text{F}$	Negation laws

Semantics of Propositional Formulas

- A formula defines a function from truth assignments to truth values
 - A truth assignment gives a truth value for each variable
- Question:
 - Why does this justify substituting one subformula by another equivalent subformula within a larger formula and then claiming the result is equivalent to the starting formula?

Exercises

- Section 1.2: 9, 11
- Give a propositional formula that is equivalent to $\phi \rightarrow \psi$, but that does not use \rightarrow
- Draw an expression tree for $p \rightarrow q \vee \neg r \wedge t$
- Which of the following are propositional formulas?
 - pq
 - $((p))$
 - $q \vee \neg \neg \neg r$
 - $p \vee \rightarrow q$

Precedence (Review)

- Should $\neg q \rightarrow \neg p$ be interpreted as
 - $(\neg q) \rightarrow (\neg p)$, or as
 - $\neg(q \rightarrow (\neg p))$?
- Precedence gives rules for implicit parentheses

Op	Prec
\neg	1
\wedge	2
\vee	3
\rightarrow	4
\leftrightarrow	5

Syntax of Propositional Formulas (Review)

- Definition of *propositional formula*:
 - A propositional variable p is a propositional formula
 - The constants **T** and **F** are propositional formulas
 - If ϕ and ψ are propositional formulas, then the following are also propositional formulas:
 - (ϕ)
 - $\neg\phi$
 - $\phi \wedge \psi$
 - $\phi \vee \psi$
 - $\phi \rightarrow \psi$
 - $\phi \leftrightarrow \psi$
 - $\phi \oplus \psi$

Expression Trees (Review)

- Example: $((a \rightarrow t) \wedge a) \wedge \neg t$

