Discrete Mathematical Structures
CS 3233 Lecture Seven

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Business

- Read Sections 2.3 for Tuesday
- Recall **Homework Three**
  Due Tuesday September 19:
  - Section 1.6: 2 (natural-language proof), 12, 18
  - Section 1.7: 4, 6, 8
  - Section 2.1: 2a, 2b, 6, 8, 18
  - Section 2.2: 2, 6, 10, 20, 26a
- Questions???

Sets

- Def: a *set* is an unordered collection of objects
  - Objects can themselves be sets
  - We will work with “naïve set theory,” in which the domain of objects is not carefully constructed, but is taken to be anything one might imagine
  - This can lead to logical inconsistencies called “paradoxes”

Set Membership

- The objects in a set are called its *elements* or *members*
  - A set is said to *contain* its members
- The membership predicate symbol used with sets is $\in$
  - It takes an object and a set
  - It is written “infix,” $x \in A$, and is read “x is a member of A”
  - It yields the truth value T just in case the object is in the set

Terminology

- Set notations:
  - $\emptyset = \{\ \}$ the *empty set*
  - {a, e, i, o, u}
  - {1, 2, 3, …, 100}
  - **N** = {0, 1, 2,…}
  - **Z** = {…, -2, -1, 0, 1, 2, …} the *integers*
  - **Q** = $\{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, q \neq 0\}$
- The examples using "…" are not formal
- How many elements are in the set $\{\emptyset\}$?

Set Builder Notation

- The *set builder* notation constructs a set by stating the property or properties elements must have to be in the set.
  - E,g., $\{x \mid \Phi(x)\}$
  - E,g., {x | x is a prime number less than 100}
- The set of objects that satisfy a formula $\Phi(x)$ is called the *extension* of the formula

Relationships Between Sets

- Two sets A and B are *equal* if A and B have the same elements
- A is a *subset* of B if every element of A is also an element of B
  - Written: $A \subseteq B$
  - $\forall x (x \in A \rightarrow x \in B)$
  - If $A \subseteq B$ and $A \neq B$, then A is a *proper subset* of B, written: $A \subset B$
- Proposition (obvious theorem):
  - $\emptyset \subseteq A$ and $A \subseteq A$

Cardinality

- Def: Let S be a set. If S contains exactly *n* distinct elements, for some nonnegative integer *n*, we say S is a *finite set* and that *n* is the cardinality of S.
  - Denoted $|S| = n$
- Otherwise, S is said to be *infinite*

Power Set

- Def: Given a set S, the *power set* of S is the set of all subsets of S
  - Denoted ***P***(S)
  - ***P***(S) = { A | A ⊆ S}
- Examples
  - P({1,2}) = {∅, {1}, {2}, {1,2}}
  - P(∅) = {∅}
  - P({∅}) = {∅,{∅}}
  - P({∅,{∅}}) = ?

Cartesian Products

- Def: An *n-tuple* $(a_1, a_2, \ldots, a_n)$ is an ordered collection whose elements are indexed by the integers 1, 2, …,n
- Def: Given sets A and B, the *Cartesian product* of A and B, denoted A x B, is the set of ordered pairs:
  - A x B = {(a,b) | a ∈ A ∧ b ∈ B}
- Can generalize to n-tuples:
  - $A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i \in 1,\ldots,n\}$

Unions and Intersections

- Given sets A and B, the *union* of A and B, A∪B, is the set containing each element of A and each element of B
- Given sets A and B, the *intersection* of A and B, A ∩ B, is the set consisting of objects that are elements of both A and B
- Venn diagrams help visualize

Disjoint Sets, Set Difference

- Sets A and B are *disjoint* if A ∩ B = ∅
- The *difference* of A and B, A – B, consists of all elements of A that are not elements of B
  - Also called the *complement of* B *with respect to* A
- Given universe U and set S, the *compliment* of S, is the compliment of S with respect to U