Discrete Mathematical Structures
CS 3233 Lecture 11

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Business

- Midterm I is next Thursday 10/5/06
- Tuesday will be devoted to review
  - Be prepared with questions
- Use the following to study for exam
  - Mock exam (which was handed out and posted on course web page)
  - Practice quizzes
  - Homework exercises
- Questions???

Preferred Formulas

- Properties of function $S \subset A x B$:
  - $\forall a \in A. \exists b \in B.(a,b) \in S$ (S is total)
  - $\forall b \in B. \exists a \in A.(a,b) \in S$ (S is surjective)
  - $\forall (a_1,b_1) \in S. \forall (a_2,b_2) \in S. a_1=a_2 \rightarrow b_1=b_2$ (S is a function)
  - $\forall (a_1,b_1) \in S. \forall (a_2,b_2) \in S. b_1=b_2 \rightarrow a_1=a_2$ (S is a injective)

Countability Exercise 1

- Theorem: If A is an uncountable set and B is a countable set, A – B is uncountable
- Proof
  - Suppose for contradiction that A – B is countable
  - This means that there is a sequence that enumerates all elements of A – B
  - We can now construct a sequence that enumerates A
    - It alternates between the sequence that enumerates A – B and the sequence that enumerates B
    - This contradicts the assumption that A is uncountable
  - It follows that the assumption A – B is countable is false, hence A – B is uncountable

Countability Exercise 2

- Theorem: If A is an uncountable set and A is a subset of B, then B is uncountable
- Proof:
  - Suppose B is countable
  - This means there is a sequence that enumerates B
  - A sequence that enumerates A can be constructed by dropping the elements of B – A, yielding the desired contradiction

Uncountability of the Reals

- Theorem
  - R, the set of real numbers, is uncountable
- Proof
  - Uses Georg Cantor's *diagonalization argument*
  - Outline
    - Assume for contradiction that there is a one-to-one correspondence, f, between N and the real interval [0,1]
    - Use f to construct a real in [0,1] that has no pre-image under f
    - Idea: for each decimal place, n, in the representation of the constructed value, choose a decimal different from the n^{th} place of f(n)
    - The fact that the constructed value differs from each value assumed by f shows that f is not onto, giving the desired contradiction