Discrete Mathematical Structures
CS 3233 Lecture Four

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Business

- Turn in Homework One
- Read Section 1.5 by Thursday
- **Homework Two:** due Tuesday September 12
  - Section 1.3: 10d, 10e, 14, 24c, 24d, 32a, 32b, 44
  - Section 1.4: 2a, 2c, 4, 10, 24, 46
  - Section 1.5: 4, 8
- Questions???

Translating English into Logic

- Every student in this class has taken calculus
  - c(x) means x is in the class
  - t(x) means x has taken calculus
- Some students in this class do not like logic
  - k(x) means x likes logic
- Not all students in this class like logic

Negations of Quantified Formulas

- De Morgan's Laws for Quantifiers
  - $\neg\exists x.p(x) \equiv \forall x.\neg p(x)$
  - $\neg\forall x.p(x) \equiv \exists x.\neg p(x)$

In-Class Exercise

- Express $\neg(\forall x\ \exists y.(y<x))$ by a formula that has all the quantifiers out front

More Translating

- $\forall p.\exists g.s(g,p)$
  - p ranges over problems
  - g ranges over groups
  - s(g,p) means group g solved problem p
- Every group solved at least one problem
- Every group solved every problem
- At least one group solved a problem

Logic and the Law

- Students in this class never break more than one law at a time
  - b(p,t,l) person p breaks law l at time t
  - c(x) means x is in the class

Predicate Formulas as Specifications

- For the domain of integers, is $\forall x.(x>3)$ true or false?
  - How about $\exists x.(x>3)$?
  - Note: "$>$" is the predicate symbol here
  - $X>3$ is another way of writing $>(x,3)$
- What does $\forall x\ \forall y.(x+y = y+x)$ mean?
  - Is it true?
  - What is the predicate here?
  - In logic, "+" is called a *function symbol* (term not introduced in the text)

In Which Numeric Domains Does each of the Following Hold?

- $\forall x.\exists y.x<y$
- $\forall x.\exists y.y<x$
- $\exists x.\forall y.x\leq y$
- $\forall x.\forall z.(x<z \rightarrow \exists y.(x<y)\wedge(y<z))$

Open and Closed Formulas

- Not in text
- A formula is *closed* if all variables are in the scope of some quantifier
- Otherwise, the formula is *open*
- $\Phi(x) \equiv \forall y.x\leq y$ is open
  - (In what numeric domains does $\Phi(0)$ hold?)

Theorems and Proofs

- A *theorem* is a statement that can be shown to be true
- A proof is a demonstration that a statement is a theorem
- Methods of proof
  - Construction of truth tables
  - Use of equivalences
    - By using these alone, can prove only logical equivalences
  - More general rules of inference

Important Related Terminology

- Result: often used to mean a theorem
- Proposition: a simple theorem, often presented without proof
- Lemma: a theorem whose main utility lies in helping to prove other, more interesting theorems
- Corollary: a theorem that follows easily from another more general theorem
- Conjecture: a statement that you suspect is true but that you do not yet have a proof for