Discrete Mathematical Structures
CS 3233 Lecture 13

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Business

- Turn in Homework 7
- Homework 8 due Thursday 10/25
  - 3.1: 2, 4, 12, 18, 24
  - 3.2: 2, 4

Bubble Sort

proc bubble sort($a_1,a_2,\ldots,a_n$)
for i := 1 to n-1
for j := 1 to n-i
if $a_j > a_{j+1}$ **then** interchange a_j and a_{j+1}
{$a_1,a_2,\ldots,a_n$ is in increasing order}

- How many comparisons are performed?

Insertion Sort

proc insertion sort($a_1,a_2,\ldots,a_n$: reals with n≥2)
for j := 2 **to** n
begin
i := 1
while $a_j > a_i$
i := i+1
m := a_j
for k := 0 **to** j – i – 1
$a_{j-k} := a_{j-k-1}$
a_i := m
end {$a_1,a_2,\ldots,a_n$ is in increasing order}

Growth of Functions

- Big-O Notation
- Def: Let f, g : Z → R. We say f(x) is O(g(x)) if there are constants C and k such that |f(x)| ≤ C|g(x)| for all x > k
  - We say "f(x) is big-oh of g(x)"
- When discussing positive-valued functions, we can drop the |·|

Example

- Let $f(x) = x^2 + 2x +1$ and $g(x) = x^2$
- f(x) is O(g(x))
  - k = 1 and C = 4 (witnesses)
  - k = 2 and C = 3
- g(x) is O(f(x))
  - k = 1 and C = 1

Example

- Let $f(x) = 7x^2$ and $h(x) = x^3$
- $7x^2$ is $O(x^3)$
  - k = 1 and C = 7
- x^3 is not $O(7x^2)$
  - There is no C such that $x^3 \leq C(7x^2)$ for "large" x

Degree of Polynomial is What Matters

- Theorem:
  Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$.
  Then f(x) is $O(x^n)$

More Examples

- Sum of first n positive integers
  $1+2+\ldots+n \leq \underbrace{n+n+\ldots+n}_{n} = n^2$
  So $\sum_{i=1}^{n} i$ is $O(n^2)$
- It follows that the complexity of both bubble sort and insertion sort is $O(n^2)$

Still More Examples

- $f(n) = n!$ is the product of the first n positive integers
  $1 \cdot 2 \cdot \ldots \cdot n \leq \underbrace{n \cdot n \cdot \ldots \cdot n}_{n} = n^n$

  So n! is $O(n^n)$

Big-O and Sums

- Def: If f_1, f_2 are real-valued functions, (f_1+f_2) is the function such that that $(f_1+f_2)(x) = f_1(x)+f_2(x)$ for all x
- Theorem:
  If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$, then $(f_1+f_2)(x)$ is $O(\max(|g_1(x)|, |g_2(x)|)$
- Corollary:
  If $f_1(x)$ and $f_2(x)$ are each $O(g(x))$, then $(f_1+f_2)(x)$ is $O(g(x))$

Big-O and Products

- Def: If f_1, f_2 are real-valued functions, $(f_1 f_2)$ is the function such that that $(f_1 f_2)(x) = f_1(x) \cdot f_2(x)$ for all x
- Theorem:
  If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$, then $(f_1 f_2)(x)$ is $O(g_1(x)\, g_2(x))$