

Discrete Mathematical Structures CS 3233 Lecture 19

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Business

- Read section 5.1
- Homework due Thursday 15 November
– 5.1: 8, 12, 14, 16

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Counting

- We are beginning Chapter 5 today
- Basic question: how can we determine how many configurations or procedures are possible in a given system?

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Product Rule

- Suppose a system or a procedure can be broken down in to two components. If there are n_1 possible ways of configuring the first component, and n_2 ways of configuring the second, then the overall system has $n_1 n_2$ configurations

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Examples

- 32 computers with 24 ports each have a total of 768 ports
- Number of bit strings of length 7 is 128
- License plates
- One to one functions
- Nested for loops
- Subsets of a finite set

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The Sum Rule

- Given two disjoint sets, one of size n_1 , the other of size n_2 , the number of ways to choose one element from one of the two sets is $n_1 + n_2$

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Examples

- How many ways are there to choose a character if upper and lowercase alphabets and numbers are all allowed?

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Counting Example

- How many passwords if they must be 8 alphanumeric characters long and must include at least one numeric?
 - Idea: all 8 character strings minus the ones that contain no numerics

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Principle of Inclusion-Exclusion

- $|A \cup B| = |A| + |B| - |A \cap B|$
- Example
 - How many bit strings of length 8 start with a 1 or end with 00?

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Pigeonhole Principle

- If $k+1$ or more objects are placed in k boxes, there is at least one box containing two or more objects
- Proof
 - If each box contains at most 1 object, together they can contain at most k objects

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Generalized Pigeonhole Principle

- If N objects are placed into k boxes, at least one box contains at least $\lceil N/k \rceil$
- Proof: Suppose each box contains at most $\lceil N/k \rceil - 1$ objects. This doesn't account for all the objects:
$$k(\lceil N/k \rceil - 1) < k((N/k + 1) - 1) = N$$
- Example
 - Among 100 people there are at least $\lceil 100/12 \rceil = 9$ people born in the same month

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