

Discrete Mathematical Structures CS 3233 Lecture 21

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Business

- Read section 5.3
- Kallen collects homework due today, Tuesday 27 November
 - 5.1: 24
 - 5.2: 2, 4, 8
- Homework due Thursday 29 November
 - 5.3: 12, 16, 24
 - Late papers will be accepted—do your best

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Counting Examples

- How many strings of lowercase letters have length 4 or less?
- How many strings of ascii characters contain “@” at least once?
- How many strings of 3 decimal digits
 - Do not contain the same digit 3 times?
 - Begin with an even digit?
 - Have exactly two digits that are 4s?

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Pigeonhole Principle

- If $k+1$ or more objects are placed in k boxes, there is at least one box containing two or more objects
- Proof
 - If each box contains at most 1 object, together they can contain at most k objects

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Generalized Pigeonhole Principle

- If N objects are placed into k boxes, at least one box contains at least $\lceil N/k \rceil$
- Proof: Suppose each box contains at most $\lceil N/k \rceil - 1$ objects. This doesn't account for all the objects:
$$k(\lceil N/k \rceil - 1) < k((N/k + 1) - 1) = N$$
- Example
 - Among 100 people there are at least $\lceil 100/12 \rceil = 9$ people born in the same month

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Pigeonhole Examples

- Drawer contains 12 brown and 12 black socks
 - How many must be selected at random to ensure having two socks of the same color?
 - How many must be selected at random to ensure having two black socks?
- Use the pigeonhole principle to show that if $|A| < |B|$, then no $f:A \rightarrow B$ is surjective

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Permutations

- Begin Section 5.3
- Definition
 - Given a set S , an r -permutation of S is an ordered arrangement of r distinct elements of S
- Theorem
 - Given a set S of size n , the number of r -permutations of S is $P(n,r) = n(n-1)(n-2)\dots(n-r+1) = n!/(n-r)!$

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Examples

- The number of alphabetic strings of length 3 consisting of distinct characters
- The number of one-to-one functions from a set of size r to a set of size n

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Combinations

- Definition
 - Given a set S , an r -combination of S is an **unordered** arrangement of r distinct elements of S

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Examples

- The number of sets of size three consisting of (distinct) alphabetic characters
- The number of subsets of size r drawn from a set of size n
 - Compared to the set of one-to-one functions from a set of size r to a set of size n , we are considering only the range of the function, not its individual values

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A Way of Thinking

- How many ways are there to order a set of size n ?
- If you only care about the first r places in the ordering, $(n-r)!$ of the orderings are effectively the same
 - This is because once I've chosen the first r places, there remain $(n-r)$ elements whose order I don't care about
 - Thus, the number of permutations is $n!/(n-r)!$
 - For the number of combinations, you also don't care about the ordering of the elements in the first r places, so you divide the number of permutations by the number of ways of ordering the first r places, which is $r!$

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