Discrete Mathematical Structures
CS 3233 Lecture 22

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Business

- Review session Tues 4 Dec 1-2:30pm
- Read section 6.1
- Practice homework (will not be collected)
  – 6.1: 5, 7, 21, 37
- Recall that final exam is in regular lecture room Wed 5 Dec 10:30-1pm

Permuations

- Begin Section 5.3
- Definition
  – Given a set S, an *r-permutation of S* is an ordered arrangement of r distinct elements of S
- Theorem
  – Given a set S of size n, the number of r-permutations of S is P(n,r) = n(n-1)(n-2)…(n-r+1) = n!/(n-r)!

Examples

- The number of alphabetic strings of length 3 consisting of distinct characters
- The number of one-to-one functions from a set of size r to a set of size n

Combinations

- Definition
  – Given a set S, an *r-combination of S* is an **unordered** arrangement of r distinct elements of S
- Theorem
  – Given a set S of size n, and an integer r, 0 ≤ r ≤ n, the number of r-combinations of S is C(n,r) = n!/((n-r)!r!)
- C(n,r) is a *binomial coeficient*

Examples

- The number of sets of size three consisting of (distinct) alphabetic characters
- The number of subsets of size r drawn from a set of size n
  – Compared to the set of one-to-one functions from a set of size r to a set of size n, we are considering only the range of the function, not its individual values

A Way of Thinking

- How many ways are there to order a set of size n?
- If you only care about the first r places in the ordering, (n-r)! of the orderings are effectively the same
  - This is because once I've chosen the first r places, there remain (n-r) elements whose order I don't care about
  - Thus, the number of permutations is n!/(n-r)!
  - For the number of combinations, you also don't care about the ordering of the elements in the first r places, so you divide the number of permutations by the number or ways of ordering the first r places, which is r!

6.1: Discrete Probability

- An *experiment* yields one of a given set of possible outcomes
- The *sample space* S is the set of possible outcomes
- An *event* E is a subset of the sample space
- Assuming each outcome in S is equally likely, the *probability* of event E is p(E) = |E|/|S|
  - This is just the sum of the probabilities of each outcome that belongs to the event

Simple Probability Example

- An urn contains 4 blue balls and 5 red balls. If one ball is chosen at random, what is the probability that it is blue?
- Rolling two dice, what is the probability of rolling a 7?
- Lottery: what is the probability of correctly choosing a set of 6 positive integers ≤ 40?
- What is the probability of being dealt 4 of a kind in a 5-card poker hand?

Combinations of Events

- Given an event $E \subseteq S$, $\bar{E}$ is the complementary event, S-E.
  $p(\bar{E}) = 1 - p(E)$
  - If a coin is flipped 10 times, what is the probability of getting heads at least once?
- If E_1 and E_2 are events in S then what is $p(E_1 \cup E_2)$?
  - What's the probability that a positive integer ≤ 100 and selected at random is divisible by neither 2 nor 5?

Probability Distributions

- What happens when all outcomes are not equally likely? Distribution is not *uniform*.
  - Given a finite sample space $S = \{x_1, \ldots, x_n\}$, a real-valued function $p: S \to [0,1]$ is a *probability distribution* if
    1. $0 \le p(x_i) \le 1$, for i = 1,2,…,n
    2. $\sum_{0 \le i \le n} p(x_i) = 1$
- Biased-coin example: heads twice as likely as tails
- Probability of event E: $p(E) = \sum_{x \in E} p(x)$