

Discrete Mathematical Structures CS 3233 Lecture One

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Course Introduction

- See syllabus:
 - <http://www.cs.utsa.edu/~winsboro/teaching/CS3233F2007/Syllabus.htm>
- Attendance is mandatory in lecture and recitation
- Read the text book
 - Lectures are associated with sections in the text as indicated on the syllabus
- If you cannot make my office hours, please set up an appointment
 - Email, call, drop by
- Please speak up!

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Section 1.1: Introduction to Logic

- Aim: formalize mathematical argument
- *Propositional logic* (also called *propositional calculus*)
 - Deals with propositions
 - A proposition is a declarative sentence that is either *true* or *false*, but not both

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Propositions

- Some sentences that are propositions
 - $1+1 = 2$
 - Today is Saturday
 - Will is your professor and San Antonio is the center of the universe
- Some sentences that are not propositions
 - Is this a CS class?
 - Not declarative
 - Read your syllabus carefully
 - Not declarative
 - $x + 5 = 7$
 - Neither true or false, since the truth value depends on the value assigned to x

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Negation

- Def: Given a proposition p , $\neg p$ denotes the negation of p
 - $\neg p$ means "it is not the case that p "
- Truth table for $\neg p$:

p	$\neg p$
T	F
F	T

Given truth assignments for p

Resulting truth value of $\neg p$

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Conjunction

- Def: Given propositions p and q , $p \wedge q$ denotes the conjunction of p and q
 - $p \wedge q$ means " p and q "
- Truth table for $p \wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Given truth assignments

Resulting truth value

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Disjunction

- Def: Given propositions p and q , $p \vee q$ denotes the disjunction of p and q
 - $p \vee q$ means "p or q" (*inclusive or*)
- Truth table for $p \vee q$:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Given truth assignments Resulting truth value

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Implication

- Def: Given propositions p and q , $p \rightarrow q$ is an implication
 - $p \rightarrow q$ means "p implies q"
 - p is the *hypothesis, antecedent or premise*
 - q is the *conclusion, consequence, or consequent*
 - Truth table for $p \rightarrow q$:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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Understanding Implications

- Some readings of $p \rightarrow q$:
 - if p , then q
 - q if p
 - q when p
 - p only if q
 - q follows from p
 - q is a necessary condition for p
 - p is a sufficient condition for q
 - q is necessary for p
 - p is sufficient for q

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Implications Related to $p \rightarrow q$

- Contrapositive:** $\neg q \rightarrow \neg p$
- Converse:** $q \rightarrow p$
- Inverse:** $\neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	T	T

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Exclusive Or

- Def: Given propositions p and q , $p \oplus q$ denotes the exclusive or of p and q
 - $p \oplus q$ means "p or q, but not both"
- Truth table for $p \oplus q$:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

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Biconditionals

- Def: Given propositions p and q , $p \leftrightarrow q$ is a *biconditional*
 - $p \leftrightarrow q$ means "p if and only if q"
- Truth table for $p \leftrightarrow q$:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Translating English to Logic

- Fred can access the wireless network only if Fred has paid his tuition
 - Let a represent “Fred can access wireless”
 - Let t represent “Fred has paid his tuition”
 - $a \rightarrow t$

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Syntax of Propositional Formulas

- Definition of *propositional formula*:
 - A propositional variable p is a propositional formula
 - The constants **T** and **F** are propositional formulas
 - If ϕ and ψ are propositional formulas, then the following are also propositional formulas:
 - (ϕ)
 - $\neg\phi$
 - $\phi \wedge \psi$
 - $\phi \vee \psi$
 - $\phi \rightarrow \psi$
 - $\phi \leftrightarrow \psi$
 - $\phi \oplus \psi$

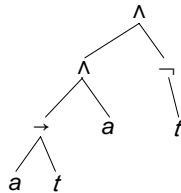
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Expression Trees

- Example: $((a \rightarrow t) \wedge a) \wedge \neg t$



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Precedence

- Should $\neg q \rightarrow \neg p$ be interpreted as
 - $(\neg q) \rightarrow (\neg p)$, or as
 - $\neg(q \rightarrow (\neg p))$?
- Precedence gives rules for implicit parentheses

Op	Prec
\neg	1
\wedge	2
\vee	3
\rightarrow	4
\leftrightarrow	5

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Semantics of Propositional Formulas

- A formula defines a function from truth assignments to truth values
 - A truth assignment gives a truth value for each variable

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Implications Related to $p \rightarrow q$

- *Contrapositive*: $\neg q \rightarrow \neg p$
- *Converse*: $q \rightarrow p$
- *Inverse*: $\neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	T	T

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Consistency

- Intuitive requirement: specifications should not contain conflicting requirements
- Definition: A collection of propositional formulas is *consistent* if there is a truth assignment that makes each formula true

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Example Inconsistent Spec

- Specification:
 - Fred can access the wireless network only if Fred has paid his tuition ($a \rightarrow t$)
 - Fred can access the wireless network (a)
 - Fred has not paid his tuition ($\neg t$)

a	t	$a \rightarrow t$	$\neg t$	$(a \rightarrow t) \wedge a \wedge \neg t$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

- $(a \rightarrow t) \wedge a \wedge \neg t$ is a *contradiction*

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A More Concise Truth Table

$((a \rightarrow t) \wedge a) \wedge \neg t$	t
T	T
T	F
F	T
F	F

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A More Concise Truth Table

$((a \rightarrow t) \wedge a) \wedge \neg t$	t
T	T
T	F
F	T
F	F

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A More Concise Truth Table

$((a \rightarrow t) \wedge a) \wedge \neg t$	t
T	T
T	F
F	T
F	F

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A More Concise Truth Table

$((a \rightarrow t) \wedge a) \wedge \neg t$	t
T	T
T	F
F	T
F	F

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Tautologies and Contradictions

- A compound proposition that is true for all truth assignments is a *tautology*
 - E.g., $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- One that is false for all assignments is a *contradiction*
- Given propositional formulas p and q , if the biconditional $p \leftrightarrow q$ is a tautology, then p and q are *logically equivalent*
 - In this case we write $p \equiv q$
 - e.g., $(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$
 - Note that \equiv is not a logical connective (i.e., a logical operator), so $p \equiv q$ is not a compound proposition
 - \equiv is part of the *meta-language* we use for discussing formulas in the *object language* of propositional calculus

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Some Important Logical Equivalences

$\phi \wedge \mathbf{T} \equiv \phi$ $\phi \vee \mathbf{F} \equiv \phi$	Identity laws
$\phi \vee \mathbf{T} \equiv \mathbf{T}$ $\phi \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$\phi \vee \phi \equiv \phi$ $\phi \wedge \phi \equiv \phi$	Idempotency laws
$\neg(\neg \phi) \equiv \phi$	Double negation law
$\phi \vee \psi \equiv \psi \vee \phi$ $\phi \wedge \psi \equiv \psi \wedge \phi$	Commutativity laws

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More Equivalences

$(\phi \vee \psi) \vee \theta \equiv \phi \vee (\psi \vee \theta)$ $(\phi \wedge \psi) \wedge \theta \equiv \phi \wedge (\psi \wedge \theta)$	Associativity laws
$\phi \vee (\psi \wedge \theta) \equiv (\phi \vee \psi) \wedge (\phi \vee \theta)$ $\phi \wedge (\psi \vee \theta) \equiv (\phi \wedge \psi) \vee (\phi \wedge \theta)$	Distributivity laws
$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$ $\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	De Morgan's laws
$\phi \vee (\phi \wedge \psi) \equiv \phi$ $\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption laws
$\phi \vee \neg\phi \equiv \mathbf{T}$ $\phi \wedge \neg\phi \equiv \mathbf{F}$	Negation laws

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