Discrete Mathematical Structures
CS 3233 Lecture Two

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Business

- Questions???
- Read Sections 1.1 by Thursday
- Homework due Thursday 8/30
  - Section 1.1: 6d, 6e, 6g, 10, 26, 28a-d
  - Hand in at beginning of class. Work alone.
- Your TA in this course is Wanying Zhao, who you can call Kallen
  - Her email is kallenzwy@hotmail.com
- Recall that attendance is required
  - This goes for recitation, too.

Implications Related to p → q

- *Contrapositive*: ¬q → ¬p
- *Converse*: q → p
- *Inverse*: ¬p → ¬q

p	q	¬p	¬q	p → q	¬q → ¬p	q → p	¬p → ¬q
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	T	T

Exclusive Or

- Def: Given propositions p and q, p ⊕ q denotes the exclusive or of p and q
  - p ⊕ q means "p or q, but not both"
- Truth table for p ⊕ q:

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

Biconditionals

- Def: Given propositions p and q, p ↔ q is a *biconditional*
  - p ↔ q means "p if and only if q"
- Truth table for p ↔ q:

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Translating English to Logic

- Fred can access the wireless network only if Fred has paid his tuition
  - Let *a* represent "Fred can access wireless"
  - Let *t* represent "Fred has paid his tuition"

Translating English to Logic

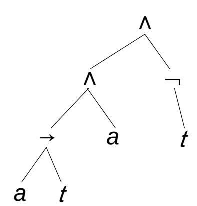
- Winsborough will not finish his reviewing on time unless he hurries
  - Let *h* represent “Winsborough hurries”
  - Let *f* represent “Winsborough finishes on time”
- Is this statement true when Winsborough is already late?

Syntax of Propositional Formulas

- Definition of *propositional formula:*
  - A propositional variable *p* is a propositional formula
  - The constants **T** and **F** are propositional formulas
  - If ϕ and ψ are propositional formulas, then the following are also propositional formulas:
    - (ϕ)
    - $\neg\phi$
    - $\phi \wedge \psi$
    - $\phi \vee \psi$
    - $\phi \rightarrow \psi$
    - $\phi \leftrightarrow \psi$
    - $\phi \oplus \psi$

Expression Trees

- Example: $((a \rightarrow t) \wedge a) \wedge \neg t$



Precedence

- Should $\neg q \rightarrow \neg p$ be interpreted as
  - $(\neg q) \rightarrow (\neg p)$, or as
  - $\neg(q \rightarrow (\neg p))$?
- Precedence gives rules for implicit parentheses

Op	Prec
¬	1
∧	2
∨	3
→	4
↔	5

A More Concise Truth Table

((*a*	→	*t*)	∧	*a*)	∧	¬	*t*
T		T		T			T
T		F		T			F
F		T		F			T
F		F		F			F

A More Concise Truth Table

((*a*	→	*t*)	∧	*a*)	∧	¬	*t*
T	T	T		T		F	T
T	F	F		T		T	F
F	T	T		F		F	T
F	T	F		F		T	F

A More Concise Truth Table

$((a$	$\rightarrow$	$t)$	$\wedge$	$a)$	$\wedge$	$\neg$	t
T	T	T	T	T		F	T
T	F	F	F	T		T	F
F	T	T	F	F		F	T
F	T	F	F	F		T	F

A More Concise Truth Table

$((a$	$\rightarrow$	$t)$	$\wedge$	$a)$	$\wedge$	$\neg$	t
T	T	T	T	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	F	F	F	F	T
F	T	F	F	F	F	T	F

Tautologies and Contradictions

- A compound proposition that is true for all truth assignments is called a *tautology*
  - *E.g.*, $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- One that is false for all assignments is a *contradiction*
- Given propositional formulas ϕ and ψ, if the biconditional $\phi \leftrightarrow \psi$ is a tautology, then ϕ and ψ are *logically equivalent* (this is an alternate definition)
  - In this case we write $\phi \equiv \psi$
    - e.g., $(\phi \rightarrow \psi) \equiv (\neg\psi \rightarrow \neg\phi)$
  - Note that $\equiv$ is not a logical connective (*i.e.*, a logical operator), so $\phi \equiv \psi$ is not a (compound) proposition
  - $\equiv$ is part of the *meta-language* we use for discussing formulas in the *object language* of propositional calculus

Some Important Logical Equivalences

$\phi \wedge \mathbf{T} \equiv \phi$ $\phi \vee \mathbf{F} \equiv \phi$	Identity laws
$\phi \vee \mathbf{T} \equiv \mathbf{T}$ $\phi \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$\phi \vee \phi \equiv \phi$ $\phi \wedge \phi \equiv \phi$	Idempotency laws
$\neg(\neg\phi) \equiv \phi$	Double nagation law
$\phi \vee \psi \equiv \psi \vee \phi$ $\phi \wedge \psi \equiv \psi \wedge \phi$	Commuitivity laws

More Equivalences

$(\phi \vee \psi) \vee \theta \equiv \phi \vee (\psi \vee \theta)$ $(\phi \wedge \psi) \wedge \theta \equiv \psi \wedge (\phi \wedge \theta)$	Associativity laws
$\phi \vee (\psi \wedge \theta) \equiv (\phi \vee \psi) \wedge (\phi \vee \theta)$ $\phi \wedge (\psi \vee \theta) \equiv (\phi \wedge \psi) \vee (\phi \wedge \theta)$	Distributivity laws
$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$ $\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	De Morgan's laws
$\phi \vee (\phi \wedge \psi) \equiv \phi$ $\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption laws
$\phi \vee \neg\phi \equiv \mathbf{T}$ $\phi \wedge \neg\phi \equiv \mathbf{F}$	Negation laws

Semantics of Propositional Formulas

- A formula defines a function from truth assignments to truth values
  - A truth assignment gives a truth value for each variable
- Questions:
  - What is a function?
  - Why does this justify substituting one subformula by another equivalent subformula within a larger formula and then claiming the result is equivalent to the starting formula?

Exercises

- Give a propositional formula that is equivalent to $\phi \to \psi$, but that does not use $\to$
- Draw an expression tree for $p \to q \lor \neg r \land t$
- Which of the following are propositional formulas?
  - pq
  - ((p))
  - $q \lor \neg\ \neg\ \neg r$
  - $p \lor \to q$
- Exercises 1.1: 5, 9, 25, 27