Discrete Mathematical Structures
CS 3233 Lecture Five

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Business

- I was sick last Thursday 9/6 and could not meet you
  - As stated in email to class, homework due last 9/6 can be turned in today
- Recall from the end of lecture 4 on Tuesday 9/4/07 that I had not yet covered quantifiers
  - Problems from homework 2 involving quantifiers are hence due Thursday 9/13 (see below)
- Homework 3: due Thursday September 13
  - Section 1.2: 22, 26
    - Solve these problems by using a sequence of equivalences from Table 6 (p.24) and the equivalence shown in Example 3 (p.23)
  - Section 1.3: 24c, 24d, 32a, 32b, 44

Universe of Discourse and Quantifiers

- The *universe of discourse* or *domain* is the set of all possible values for variables
- We can refer to values in the universe either by using constant symbols (like "Fred") or by using quantifiers
- There are two quantifiers in standard predicate calculus: *for all* ($\forall$) and *there exists* ($\exists$)
- There are called the universal quantifier and the existential quantifier, respectively

Universal Quantifiers

- The universal quantification of p(x) is the following proposition:
  - "p(x) is true for all values of x in the universe of discourse"
  - Written $\forall x\ p(x)$ or $\forall x.p(x)$
- Similarly, if $\phi(x)$ is a formula in x, $\forall x.\phi(x)$ means the formula holds for all elements of the universe
  - What does $\forall x.(r(x) \rightarrow q(x))$ mean?
  - How is it different from $(\forall x.r(x)) \rightarrow (\forall x.q(x))$?

Universal Quantifiers

- Note that $\forall x.p(x) \equiv \forall y.p(y)$
- If the universe of discourse is {0, 1, 2}, then $\forall x.p(x) \equiv p(0) \wedge p(1) \wedge p(2)$
- Can you always rewrite $\forall x.p(x)$ this way?
  - What if the universe of discourse is infinite?
  - Logical statements are finite objects

Existential Quantifiers

- The existential quantification of p(x) is the following proposition:
  - "there exists an element x in the universe of discourse such that p(x) is true"
  - Written $\exists x\ p(x)$ or $\exists x.p(x)$
  - What does $\exists x.(r(x) \wedge \neg q(x))$ mean?
- If the universe of discourse is {0,1,2}, then
  - $\exists x.p(x) \equiv p(0) \vee p(1) \vee p(2)$

Scope of Quantifiers

- (∃x.x>3) ∧ (∃x.x<1)
- The scope of a quantifier is the formula following the dot

Negations of Quantified Formulas

- De Morgan's Laws for Quantifiers
  ¬∃x.p(x) ≡ ∀x.¬p(x)
  ¬∀x.p(x) ≡ ∃x.¬p(x)

Predicate Formulas as Specifications

- For the domain of integers, is ∀x.(x>3) true or false?
  - How about ∃x.(x>3) ?
  - Note: ">" is the predicate symbol here
  - X>3 is another way of writing >(x,3)
- What does ∀x ∀y.(x+y = y+x) mean?
  - Is it true?
  - What is the predicate here?
  - In logic, "+" is called a *function symbol* (term not introduced in the text)