Mock Exam for Midterm I
Discrete Mathematical Structures CS3233

The actual exam will be closed-book and will include true/false and multiple-choice questions. In the actual exam, you will write your answers on the same paper on which the questions are printed. This collection of problems is intended to be representative of the material that is important on the exam. It is longer than the actual exam, but the actual exam is a little long, too. The actual exam is also rather hard. It will be graded on a curve.

In addition to the problems given here, the problems from in-class practice quizes are also representative of the questions on the exam.

1. Define each of the following: tautology, consistent, inconsistent, valid, contradiction, logical equivalence, contrapositive, converse.

2. What are some universes of discourse in which the following are true?

   (a) $\forall x \exists y (y > x)$

   (b) $\exists x \forall y (x \neq y \rightarrow x < y)$

   (c) $\forall x \forall y ((x < y) \rightarrow (\exists z (x < z \wedge z < y)))$

3. Prove that the tautology corresponding to the *Resolution* rule of inference is true by using truth tables. (See Table 1 of Section 1.5, p.66.)

4. Write a formal, logical proof similar to the ones in Examples 6 and 7 of Section 1.5 proving q from the assumptions p and $p \vee r \rightarrow q$.

5. Write a formula that says there is no least number. Is the formula true when the universe of discourse is Z (the set of integers)? When it is R^+ (the positive reals)? When it is N (the set of natural numbers)?

6. Write a formula that is true if and only if the universe of discourse is *dense*, meaning that between any two distinct numbers there is a third distinct number.

7. Write formulas that holds if and only if $S \subseteq A \times B$ is:

   (a) A function.

   (b) Total.

8. What is the powerset of $S = \{0, 1\}$?

9. What is the definition of a *rational* number?

10. Given a finite set S of size n, how many elements are there in $\wp(S)$, the powerset of S?

11. Use the definitions of union, intersection, and set complement, as well as De Morgan's First Law of *logical* equivalence (namely, $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$) to prove De Morgan's first set identity (namely, $\overline{A \cup B} = \overline{A} \cap \overline{B}$).