Mock Exam for Midterm II
Discrete Mathematical Structures CS3233

November, 2007

1. Define $f(n) = \mathcal{O}(g(n))$, $f(n) = \Omega(g(n))$, and $f(n) = \Theta(g(n))$.
2. Prove or disprove: $(n^2 + n^3)/2 = \Theta(n^3)$.
3. Prove or disprove: $n^2 \log n + n^2 = \Theta(n^2)$.
4. What is the best big-O estimate of the number of comparisons that are performed by an algorithm that takes a list of n integers and finds the least of the first 100 values? Justify your answer.
5. What is the worst-case time complexity of finding the least value in a list of n integers? Select the one best answer from the following list: $\mathcal{O}(1), \mathcal{O}(\log n), \mathcal{O}(n), \mathcal{O}(n \log n), \mathcal{O}(n^2), \mathcal{O}(n^3), \mathcal{O}(2^n)$?
6. What is the worst-case time complexity of using binary search to find determine whether a given value is in a given sorted list of integers? Assume that the time required to obtain the sorted list as input is negligible, as if, say, it were already available in memory. Select the one best answer from the following list: $\mathcal{O}(1), \mathcal{O}(\log n), \mathcal{O}(n), \mathcal{O}(n \log n), \mathcal{O}(n^2), \mathcal{O}(n^3), \mathcal{O}(2^n)$?
7. Use mathematical induction to prove that $n^3 - n$ is divisible by 3 for all natural numbers n
8. Use induction to show that $P(n) \equiv \sum_{1 \leq i \leq n} (-1)^{i-1} i^2 = (-1)^{n-1} n(n+1)/2$ holds for all positive integers n.
9. Is the set of negative integers well ordered? Why or why not?
10. Is the set of integers greater than 100 well ordered? Why or why not?
11. Determine whether the following are valid recursive definitions of a function $f : N \to Z$:
    (a) Valid or invalid: $f(0) = 0, f(1) = 1, f(n) = 2f(n-2)$ for $n > 1$
    (b) Valid or invalid: $f(0) = 0, f(1) = 1, f(n) = 2f(n)$ for $n > 1$
    (c) Valid or invalid: $f(0) = 0, f(1) = 1, f(n) = 2f(n+1) + f(n+2)$ for $n > 1$