Discrete Mathematical Structures
CS 3233 Lecture Three

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Business

- Questions???
- Recall: Homework 2 due Thursday January 24
  - Section 1.2: 2, 6, 10
    - I am removing problem 12
  - Section 1.3: 10d, 10e, 14, 24c, 24d, 32a, 32b, 44
- Practice quiz 1
  - Quizzes are not turned in and do not affect your grade

Universe of Discourse and Quantifiers

- The *universe of discourse* or *domain* is the set of all possible values for variables
- We can refer to values in the universe either by using constant symbols (like "Fred") or by using quantifiers
- There are two quantifiers in standard predicate calculus: *for all* ($\forall$) and *there exists* ($\exists$)
- There are called the universal quantifier and the existential quantifier, respectively

Universal Quantifiers

- The universal quantification of p(x) is the following proposition:
  - "p(x) is true for all values of x in the universe of discourse"
  - Written $\forall x\ p(x)$ or $\forall x.p(x)$
- Similarly, if $\phi(x)$ is a formula in x, $\forall x.\phi(x)$ means the formula holds for all elements of the universe
  - What does $\forall x.(r(x) \rightarrow q(x))$ mean?
  - How is it different from $(\forall x.r(x)) \rightarrow (\forall x.q(x))$?

Existential Quantifiers

- The existential quantification of p(x) is the proposition
  - "there exists an element x in the universe of discourse such that p(x) is true"
  - Written $\exists x\ p(x)$ or $\exists x.p(x)$
  - What does $\exists x.(r(x) \wedge \neg q(x))$ mean?
- If the universe of discourse is {0,1,2}, then
  - $\exists x.p(x) \equiv p(0) \vee p(1) \vee p(2)$

Scope of Quantifiers

- $(\exists x.x>3) \wedge (\exists x.x<1)$
- The scope of a quantifier is the formula following the dot

Universal Quantifiers

- Note that $\forall x.p(x) \equiv \forall y.p(y)$
- If the universe of discourse is {0, 1, 2}, then $\forall x.p(x) \equiv p(0) \wedge p(1) \wedge p(2)$
- Can you always rewrite $\forall x.p(x)$ this way?
  - What if the universe of discourse is infinite?
  - Logical statements are finite objects

Negations of Quantified Formulas

- De Morgan's Laws for Quantifiers
  - $\neg\exists x.p(x) \equiv \forall x.\neg p(x)$
  - $\neg\forall x.p(x) \equiv \exists x.\neg p(x)$

Name: _____

Quiz 1/22/08
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1. How can you tell whether a given string is a propositional formula?

2. What is the syntax of propositional formulas?

3. What is the semantics of a propositional formula?

4. What does it mean to say that two formulas are equivalent?

5. If a propositional formula Ψ contains a subformula ψ, ψ is equivalent to ϕ, and Φ is derived from Ψ by replacing ψ by ϕ, explain why Φ is equivalent to Ψ.