Discrete Mathematical Structures
CS 3233 Lecture Five

Prof. William Winsborough
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Business

- Questions???
- **Recall:** Homework 3 due Thursday 1/31:
  - Section 1.4: 2a, 2c, 4c-f, 10 (except h), 24a, 24d, 46
    - Work 4a, 4b, 10h together
    - In 46, ***explain your answers*** and assume the usual interpretation for arithmetic and relational operators
  - Section 1.5: 4, 8, 16
  - Section 1.6: 2 (natural-language proof), 12, 18
  - Section 1.7: 4, 6, 8

Practice Exercises

- Section 1.4: 4a, 4b, 10h

Rules of Inference for Propositional Logic

- A general, systematic method of proving formulas
- See Table 1 p.66
  - Known equivalences can also be used in proofs
- Use rules of inference to show
  - These hypotheses:
    - If it does not rain or if it is not foggy, the sailing race will be held and the lifesaving demonstration will take place
    - If the race is held, the trophy will be awarded
    - The trophy was not awarded
  - Imply this conclusion:
    - It rained

Rules of Inference for Universally Quantified Statements

- Universal instantiation

$$\frac{\forall x.\ p(x)}{p(c)}$$

  - For any c
- Universal generalization

$$\frac{p(c)}{\forall x.\ p(x)}$$

  - Must show p(c) for arbitrary c

Rules of Inference for Existentially Quantified Statements

- Existential instantiation

$$\frac{\exists x.\ p(x)}{p(c)}$$

  - c must be a new name (constant) that does not appear earlier in the proof
- Existential generalization

$$\frac{p(c)}{\exists x.\ p(x)}$$

  - c can be any name

Universal Modus Ponens

- Combines propositional modus ponens with universal instantiation:

$$\frac{\forall x.\Phi(x) \rightarrow \Psi(x) \quad \Phi(c)}{\Psi(c)}$$

Practice Exercises

- Section 1.5: 3, 13, 15

Some Important Methods of Proof

- Direct proof of $p \rightarrow q$
  - Assume p, derive q
- Proof by Contraposition:
  directly prove the contrapositive $\neg q \rightarrow \neg p$
  - Assume $\neg q$, derive $\neg p$
- Vacuous and trivial proofs
  - Show that $p \rightarrow q$ holds by showing that p does not hold
- Proof by contradiction
  - Prove p by directly proving $\neg p \rightarrow F$

Practice Exercises

- Section 1.6: 1 (natural-language proof), 11, 13, 17

Proof Methods for Quantifiers

- Existence proofs (p91)
  - Constructive: find a witness
  - Nonconstructive: Use case analysis and the tautology $p \vee \neg p$

Practice Exercises

- Section 1.7: 3, 5, 7