Discrete Mathematical Structures
CS 3233 Lecture Six

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Business

- Homework 3 may be turned in Thursday 2/7
  - I didn't get a chance to do any proofs by case analysis or to talk about constructive and non-constructive proofs
  - For reference, the assignment was:
    - Section 1.4: 2a, 2c, 4c-f, 10 (except h), 24a, 24d, 46
      - Work 4a, 4b, 10h together
      - In 46, ***explain your answers*** and assume the usual interpretation for arithmetic and relational operators
    - Section 1.5: 4, 8, 16
    - Section 1.6: 2 (natural-language proof), 12, 18
    - Section 1.7: 4, 6, 8
- **Homework 4**, due Thursday 2/7
  - Section 2.1: 2a, 2b, 6, 8, 18
- Read Sections 2.1, 2.2, 2.3
- Questions???

Proof Methods for Quantifiers

- Existence proofs (p91)
  - Constructive: find a witness
  - Nonconstructive: Use case analysis and the tautology $p \vee \neg p$

Practice Exercises

- Section 1.7: 3, 5, 7

Sets

- $s \in S$ means s is an *element* of set S
- Given sets A and B,
  - $A \subseteq B$ means A is a *subset* of B
    - This means $x \in A$ implies $x \in B$
  - $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
  - $A \subset B$ means A is a *proper subset* of B
    - $A \neq B$
  - A = { $a_1, a_2, \ldots$ } means that A is enumerated by the sequence $a_1, a_2, \ldots$
- *Set comprehension* : the set of values that satisfy a given property
  - Also called "set builder" notation
  - *E.g.,* EvenInts = $\{ x \mid x \in \mathbb{Z} \wedge \exists z \in \mathbb{Z}. x=2z \}$

Basic Operators

- Empty set
  - $\varnothing = \{ \}$
- Union
  - $A \cup B = \{ x \mid x \in A \vee x \in B \}$
- Intersection
  - $A \cap B = \{ x \mid x \in A \wedge x \in B \}$
- Set difference
  - $A - B = \{ x \mid x \in A \wedge \neg x \in B \}$
- Venn diagrams
- Universe U is the set consisting of all objects under consideration
  - Resembles a universe of discourse
- Compliment
  - $\overline{A} = U - A$
- Cartesian product:
  - $A \times B = \{ (a,b) \mid a \in A \wedge b \in B \}$

Identities and Big Unions, Intersections

- Set Identities
  - Review Table 1, page 124
  - Review Example 11, page 125
- Given a collection of sets $A_1, A_2, \cdots, A_n$
  - $\bigcup_{1 \le i \le n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$
  - $\bigcap_{1 \le i \le n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$

Introduction to Functions

- Given sets A and B, a *function from* A *to* B is an assignment of exactly one element of B to each element of A
- For $a \in A$, we write f(a) = b if $b \in B$ is the unique element associated with a by f
- We write $f : A \to B$ to indicate that f is a function from A to B
  - $A \to B$ is called the *type* of f

Function Graphs

- This is a different usage of "graph" than either of these:
  - The plot of f(x) on the Cartesian plane
  - A set of nodes and a set of edges
- Definition
  - Given a function $f : A \to B$, the *graph* of f is the set of ordered pairs $\{(a,b) \mid a \in A \text{ and } b = f(a)\}$
- Given two sets, A and B, A *binary relation* r between A and B is a subset $r \subseteq AxB$
- Formally speaking, a function <u>**is**</u> its graph
  - Thus, f is a subset of A x B (*i.e.,* a binary relation) in which each element of A occurs exactly once

Formal Definition of Function

- A binary relation $r \subseteq AxB$ is a *function* if
  - $\forall b \forall a_1 \forall\, a_2.((a_1,b) \in r \wedge (a_2,b) \in r) \to a_1 = a_2$
  - In this case, we write r(a) for b
- A function r is *total* (called a *total function*) if additionally
  - $\forall a \exists b.(a,b) \in r$

Terminology

- Let $f : A \to B$
  - We say f *maps* A to B
  - A is the *domain* of f
  - B is the *codomain* of f
  - If f(a) = b,
    - b is the *image* of a and
    - a is the *pre-image* of b
  - If $S \subseteq A$, the *image* of S is $\{f(s) \mid s \in S\}$ and is denoted by f(S)
  - $f(A) = \{f(a) \mid a \in A\}$ is called the *range* of f

Examples

- Successor function
  - $s : N \to N$
  - s(n) = n+1
- Floor
  - floor : $R \to Z$
  - floor(x) = greatest integer less than x
- Square root
  - $\surd : R \to C$
  - The square root of a real is a complex number
- Truth assignment
  - $\sigma : \mathcal{V} \to \{\text{true, false}\}$