Let us now explore the problem of finding the optimum tree. When \( N = 3 \), for example, let us assume that the keys \( K_1 < K_2 < K_3 \) have respective probabilities \( p, q, r \). There are five possible trees:

![Tree Diagrams](image)

Cost: \( 3p + 2q + r \quad 2p + 3q + r \quad 2p + q + 2r \quad p + 3q + 2r \quad p + 2q + 3r \)

Figure 14 shows the ranges of \( p, q, r \) for which each tree is optimum; the balanced tree is best about 45 percent of the time, if we choose \( p, q, r \) at random (see exercise 21).