4/25/12

KR, p. 496

#13. A group of n, n>2, people play "odd person out" to decide who buys drinks for the group. Each person flips a fair coin simultaneously, and the person that has an outcome different from the rest of them buys drinks. If there is no single odd person out, then the coins are flipped again. This is repeated until an odd person out is identified.

Let X be the number of times the coin-flip experiment is conducted.

(a) $P(X=1) = 2 \cdot \binom{n}{1} / 2^n$ ← total # of possible outcomes

H.=1

H or T being different

choices for the odd person

Each round is a Bernoulli Trial

p = prob. of success $= 2 \cdot \binom{n}{1} / 2^n = \frac{n}{2^{n-1}}$

(b) $P(X=k) = q^{k-1} p = \left(1 - \frac{n}{2^{n-1}}\right)^{k-1} \cdot \frac{n}{2^{n-1}}$.

$X \sim \text{Geometric}(p)$

(c) $E(X)$

4.82

mean grade in a test was 72

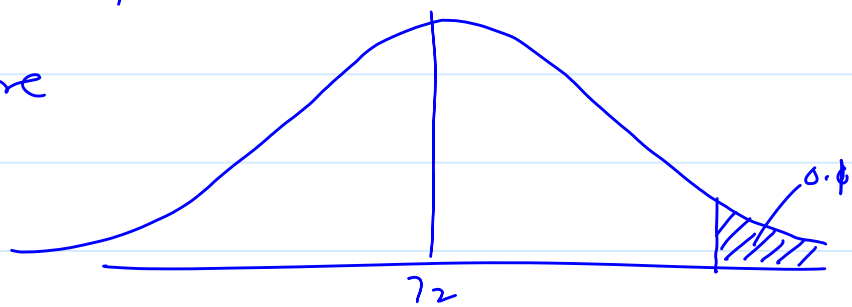
Std. dev. 9

Top 10% students receive ~~an~~ A's.

What is the min. score a student must get to secure an A.

$\mu = 72 \qquad \sigma = 9$

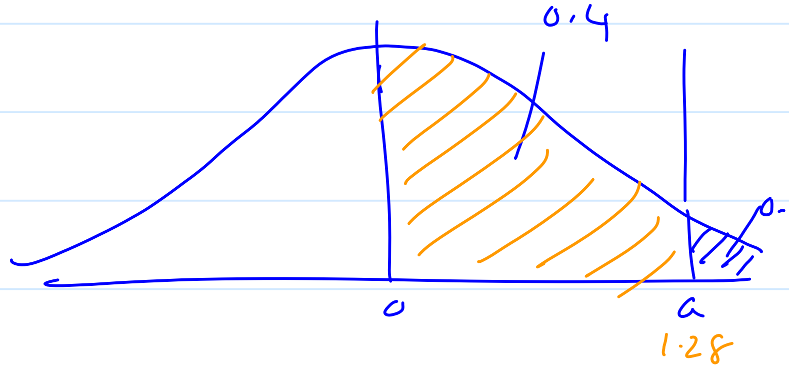
$X \sim$ Student score



$$Z = \frac{X-\mu}{\sigma} = \frac{X-72}{9}$$

is the corresponding Std. normal RV

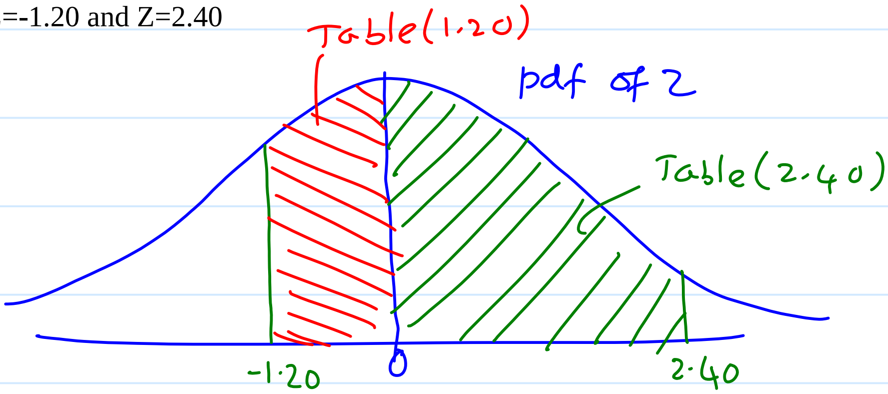
$P(Z \geq a) = 0.1$ find 'a'



$a = \text{Table}^{-1}(0.4) = 1.28$

$X = \mu + a\sigma = 72 + (1.28)9 = 72 + 10.9 = 82.9$

4.74 Find the area under the curve between Z=-1.20 and Z=2.40



$P(-1.2 \le Z \le 2.4)$

$= P(-1.2 \le Z \le 0)$
$+ P(0 \le Z \le 2.40)$

$= P(0 \le Z \le 1.2)$ (due to Symmetry)
$+ P(0 \le Z \le 2.40)$

$=$ Tableentry(1.2) + Table(2.4)
$= 0.3849 + 0.4918 = 0.8767$

4.72' Test mean score 78, $\sigma = 10$

Determine the std. score of a Student's score of 95

X = Score of a Student $\quad X \sim N(78, 10^2)$

Z = Std. Score $= \dfrac{X-\mu}{\sigma} = \dfrac{X-78}{10}$

$X = 95, \quad Z = \dfrac{95-78}{10} = 1.7$

(b') Std. Score of -0.5 What is the actual score?

$Z = -0.5 \Rightarrow X = \mu + Z\sigma = 78 + (-0.5)10 = 73$

Quick Probability Estimates

Let X be a nonnegative RV with finite mean μ.

Markov Inequality $P(X \geq k) \leq \frac{\mu}{k}, \quad k > 0$

Let X be a RV with finite mean μ and variance σ^2.

Chebyshev Inequality $P(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2}, \quad k > 0$

$\sigma^2 = V(X)$

[KR] Section 7.4, Problem 35

Use Chebyshev's inequality to find an upper bound on the prob. that the # of tails in n fair coin flips to deviates from the mean by more than $5\sqrt{n}$.

$p = \frac{1}{2}$ $\quad n \quad$ X = # of tails in n flips

$\mu = E(X) = np = \frac{n}{2} \qquad X \sim Bin(n, 1/2)$

$V(X) = npq = n \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{n}{4}$

$$P(|X-\mu| \geq 5\sqrt{n}) \leq \frac{V(X)}{(5\sqrt{n})^2} = \frac{n/4}{25n} = \frac{1}{100} = 0.01$$

[KR] Section 7.4, Page 493, Problem 38

The # of soda cans filled in a day is an average of 10000 cans with a variance of 1000 cans^2.

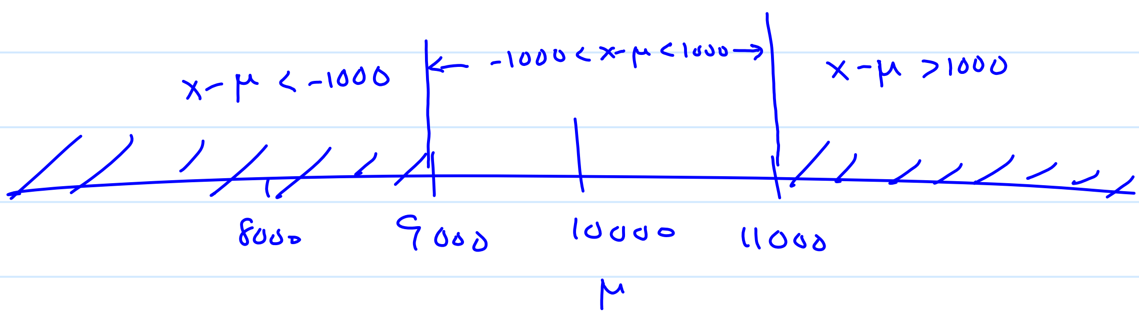
a. Estimate the probability that more than 11,000 cans are filled in a day.

X = # of cans filled in a day

$\mu = 10000 = E(X) \qquad \sigma^2 = 1000$

$$P(X \geq 11{,}000) \leq \frac{\mu}{11000} = \frac{10000}{11000} = \frac{10}{11}.$$

b'. Estimate the probability that more than 11000 cans or fewer than 9000 cans are filled in a day.



$$P(|X-\mu| \geq 1000) \leq \frac{\sigma^2}{1000^2} = \frac{1000}{1000^2} = \underline{0.001}$$

b. What is the prob. that the number of cans produced in a day is between 9,000 and 11,000?

$$P(|X-\mu| < 1000) = 1 - P(|X-\mu| \geq 1000)$$

$$\geq \underline{1 - 0.001 = 0.999}$$

Law of Large Numbers and Central Limit Theorem

Let $S_n = X_1 + X_2 + \cdots + X_n$, where

each X_i has finite mean μ and variance σ^2.

X_i are indep. and identically distributed

$$\lim_{n\to\infty} \left(\left| \frac{S_n}{n} - \mu \right| \geq \epsilon \right) = 0 \qquad \text{Law of large numbers}$$

ϵ: a small value

$$\left(\lim_{n\to\infty} \frac{S_n}{n} \right) \to \mu$$

$\dfrac{S_n - n\mu}{\sqrt{n}\,\sigma}$ is asymptotically a std. normal RV

for sufficiently large n.

Central Limit Theorem.